COMMON CORE
ALGEBRA I

VERSION 1.0

LESSONS BY KIRK WEILER

HOMEWORK SETS BY KIRK WEILER AND GARRETT MATULA

eMathInstruction.com
About the Author – Kirk Weiler has been a teacher of mathematics at Arlington High School for the past 15 years. For the past three years he has also served as the Math Department Coordinator. While at Arlington, he has taught courses ranging from Algebra 1 to Advanced Placement Calculus. He was educated as an engineer, earning both bachelors and masters degrees in engineering from the University of Illinois and Cornell University, respectively. He then left engineering for education, earning his masters in mathematics education from Syracuse University. In 2006 he earned his National Board Certification in Young and Adult Mathematics Education. From 2006 until 2008, he served as the Editor-in-chief of the Arlington Algebra Project, a collaborative effort by 26 middle school and high school teachers to write an electronic textbook for an introductory Algebra 1 course. In 2008, Kirk founded eMathInstruction and published Algebra 2 with Trigonometry. Common Core Algebra I is eMathInstruction’s second offering and the first to have complete screencast support.

Acknowledgements – I’d like to thank my wife Shana for all of the support she has given me through the process of writing this book. It took six month of nights sitting at the computer after children went to sleep, but she put up with those late nights and always believed I could finish. I’d like to sincerely thank Garrett Matula for helping out on the homework sets for early portions of the text. His help gave me the mental strength to continue on in the middle of a tough winter. I’d also like to give a shout-out to my Math 4 Honors students at Arlington during the 2013-2014 school year. They were awesome in helping me refine my screencasting techniques with suggestions that I’m still trying to process. A googolplex of thanks to Fraz Lugay for editing the answer key and removing that time drain so that I could pursue the videos during the summer of 2014.

Cover Design – I would like to especially thank Michael Frey for all of his hard work on the design of the workbook cover. More of his great work can be found at freyartanddesign.com.

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UNIT #1

THE BUILDING BLOCKS OF ALGEBRA

Lesson #1 – Rates, Patterns and Problem Solving
Lesson #2 – Variables and Expressions
Lesson #3 – The Commutative and Associative Properties
Lesson #4 – The Distributive Property
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Lesson #11 – Algebraic Puzzles
Welcome to Algebra I! Algebra at its core is all about using the properties of numbers (how they behave) to manipulate unknowns, called variables. But, in practicality, Algebra is used to recognize patterns, turn them into mathematical relationships, and then use these relationships for useful purposes. Today’s lesson, being the first of the course, is exploratory in nature and will utilize a basic understanding of rates or ratios.

Exercise #1: Answer the following rate/ratio questions using multiplication and division. Show your calculation (and keep track of your units!).

(a) If there are 12 eggs per carton, then how many eggs do we have in 5 cartons?

(b) If a car is traveling at 65 miles per hour, then how far does it travel in 2 hours?

(c) If a pizza contains 8 slices and there are 4 people eating, how many slices are there per person?

(d) If a biker travels 20 miles in one hour, how many minutes does it take per mile traveled?

Rates show up everywhere in the real world, whether it is your pay per hour of work or the texts you can send per month. Rates are all about multiplication and division because they ultimately are a ratio of two quantities, both of which are changing or varying.

Exercise #2: A runner is traveling at a constant rate of 8 meters per second. How long does it take for the runner to travel 100 meters?

(a) Experiment solving this problem by setting up a table to track how far the runner has moved after each second.

<table>
<thead>
<tr>
<th>time, ( t ) (seconds)</th>
<th>Distance, ( D ) (meters)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td></td>
</tr>
</tbody>
</table>

(b) Create an equation that gives the distance, \( D \), that the person has run if you know the amount of time, \( t \), they have been running.

(c) Now, set up and solve a simple algebraic equation based on (b), that gives the exact amount of time it takes for the runner to travel 100 meters.
The previous exercise showed how we can take a pattern and extend it into the world of algebra, a world that contains symbols and conventions that may seem strange, but hopefully somewhat familiar from previous work. In the final exercise, we will tackle a larger problem to see how rates, patterns, and algebra can combine to solve a more challenging problem.

**Exercise #3:** A man is walking across a 300 foot long field at the same time his daughter is walking towards him from the opposite end. The man is walking at 9 feet per second and the daughter is moving at 6 feet per second. How many seconds will it take them to meet somewhere in the middle?

(a) Draw a diagram to help keep track of where the man and his daughter are after 1 second, 2 seconds, 3 seconds, etcetera. Create a table as well that helps keep track of how far each one of them has traveled as time goes on.

<table>
<thead>
<tr>
<th>Time (seconds)</th>
<th>Father’s Distance (feet)</th>
<th>Daughter’s Distance (feet)</th>
<th>Total Distance (feet)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(b) What must be true about the distances the two have traveled when they meet somewhere in the middle?

(c) Create equations similar to **Exercise #3** to predict the distance the father has traveled and the distance the daughter has traveled.

(d) Create and solve an equation to predict the exact amount of time it takes for the father and daughter to meet in the middle.
**COMMON CORE ALGEBRA I HOMEWORK**

**RATES, PATTERNS AND PROBLEM SOLVING**

**FLUENCY**

1. Answer the following rate questions based on either multiplication or division. Think carefully about which is required (they will be mixed up). Show the calculation and units that you use.

   (a) A child bought 4 bags of rubber bands to make into bracelets. If there are 80 rubber bands per bag, how many total rubber bands did he buy?

   (b) Kirk has 42 pieces of candy to divide evenly between his three children. If he puts the pieces into three boxes, how many pieces of candy are there per box?

   (c) A car traveling on the Taconic parkway travels 84 miles in two hours. What is the cars speed (a special type of rate) in miles per hour?

   (d) A car salesperson earns a $500 fee per car she sells. If she sells 4 cars in one day, how much money does she earn in fees?

2. If there are 4 quarts in a gallon, and 2 pints in a quart, and 2 cups in a pint, then how many cups are in a gallon? Show your calculation or explain how you arrive at your answer.

3. A person driving along the road moves at a rate of 56 miles per hour driven. How far does the person drive in 1.5 hours? Show the calculation you use in your answer and give your answer proper units.

4. Mr. Weiler has 32 students in his class. He wishes to place them into 8 groups of equal size. Which of the following represents the number of students per group?

   (1) 256  
   (2) 2  
   (3) 6  
   (4) 4
APPLICATIONS

5. Seating in theaters or auditoriums is often arranged such that rows closer to the stage have less seats than rows farther away. An example of a seating chart for a theater is shown below.

(a) Assuming this pattern continues, fill out the following table:

<table>
<thead>
<tr>
<th>Row, $r$</th>
<th>Number of Seats, $S$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>9</td>
</tr>
<tr>
<td>2</td>
<td>11</td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td></td>
</tr>
</tbody>
</table>

(b) Jonathan tries to mathematically model the number of seats in a given row. He tries to come up with an equation for the number of seats and determines:

\[ S = 7r + 2, \text{ where } S \text{ is the number of seats in row, } r \]

Does this equation work for $r = 1$? What about for $r = 2$ and $r = 3$? Show calculations that support your yes/no answers.

(c) The correct equation is: $S = 2r + 7$. Verify this equation matches your table for $r = 1$, $r = 2$, and $r = 3$.

(d) According to the formula from part (c), how many seats are in the 15th row? Show your calculation.

(e) Finally, let’s say we know that a certain row has 91 seats in it. Which row is it? Try to set up and solve a simple equation that gives you this answer.
Algebra is the process of using the properties of numbers to manipulate unknown or changing quantities. These quantities are known as variables and are often represented using letters to distinguish them from numbers we do know (which we just use the numbers for). When we group (combine) numbers together we get what is known as an expression.

Exercise #1: Review order of operations by giving the value of each of the following purely numerical expressions. Do these without a calculator in order to review basic middle school number concepts.

(a) $3 \times 2 + 7$
(b) $8 - \frac{1}{2} \cdot 6 + 24 \div 6$
(c) $4(8 - 6) - 7(5 - 3)$

(d) $\frac{5^2 - 4^2 + 3}{1 - 5}$
(e) $(2 - 7)(5 - 3) + 3^2$
(f) $\frac{-16 + 5 \cdot 2}{2^3}$

Knowing your order of operations is absolutely essential. Once we move past expressions that contain only numbers to ones that contain variables you need to be able to “read” an expression and understand what is being done to the variable.

Exercise #2: If the letter $x$ represents some unknown quantity, explain the calculation that each of the following expressions involving $x$ represents.

(a) $3x - 8$
(b) $\frac{x - 4}{2}$
(c) $4x^2 - 8$
If you can read an algebraic expression (i.e. one that contains variables), then you should also be able to **evaluate the expression**.

**Evaluating Expressions**

Finding the results of the calculations of an expression when all variable values are known.

**Exercise #3:** For each given expression, explain in steps what the calculation is doing and then find its value for the given variable values.

(a) Evaluate $4x - 7$ when $x = 5$. First explain what calculations are occurring in the expression and then find its value.

(b) Evaluate the expression $8 - 2x^2$ when $x = -3$. Show the calculations you do and the order in which you do them:

<table>
<thead>
<tr>
<th>Calculation:</th>
<th>Explanation:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(c) Evaluate the expression $\frac{2(x + 8)}{3} + 1$ for when $x = -2$. Show the steps in your calculation.

**Exercise #4:** What is the value of the expression $\frac{1}{2}x^2 - 2x - 3$ when $x = 4$?

(1) −3          (3) 3

(2) −8          (4) 7
COMMON CORE ALGEBRA I HOMEWORK

FLUENCY

1. Using order of operations evaluate the following numerical expressions. Do not use a calculator for this section.

   (a) \(22 - 2 \cdot 6\)  \hspace{1cm} (b) \(6 - \frac{1}{4} \cdot 16 + 21 \div 3\)  \hspace{1cm} (c) \((8 - 5)(5 - 3)^2\)

2. Evaluate the following expressions for the values of \(x\) given. Show the steps in your calculation.

   (a) \(\frac{4(x - 2)}{(x - 1)}\) when \(x = 0\)  \hspace{1cm} (b) \(-\frac{3x^2 + 4}{4} - 1\) when \(x = -2\)  \hspace{1cm} (c) \(-\frac{2x + 4(x - 1)}{x^2 - 1}\) when \(x = 2\)

APPLICATIONS

3. Robert just got his first job and is saving 45 dollars a week. He also has 155 dollars saved from his birthday that just passed. To see how much money he will have in his bank account Robert came up with the following expression: \(45w + 155\), where \(w\) is the number of weeks that he has been saving.

   (a) Exactly how much will he have saved in 6 weeks?

   (b) After his first month he had more than he expected to have due to interest the bank provided. This let Rob come up with a better expression, \(\frac{w^2}{25} + 45w + 155\), where \(w\) is the number of weeks. How much will he have in 1 year?
**Reasoning**

4. Input the following two expressions into your calculator and see what you get.

(a) \((-5)^2 + 2 \times (3+1)\)  
(b) \(-5^2 + 2 \times 3 + 1\)

(c) Explain what changed from the expression in (a) to (b) and why that changed your answer.

5. Andrew received a 95 on his last test and the only question he got wrong was the following.

(a) Read through the question and Andrew’s work. Find and circle his mistake.
(b) Explain what he did wrong and what he should have done.

**Evaluate:** \(x^2 - 2(x - 3)\) when \(x = 3\).

Andrews work:
\[
= x^2 - 2(x - 3) \\
= 3^2 - 2(3 - 3) \\
= 3^2 - 2(0) \\
= 9 - 2(0) \\
= 7(0) \\
= 0
\]

(c) Using your knowledge and abilities show Andrew how to evaluate the expression correctly. State the correct value.
**THE COMMUTATIVE AND ASSOCIATIVE PROPERTIES**

**COMMON CORE ALGEBRA I**

Numbers combine through the **operations** of addition, subtraction, multiplication, and division to produce other numbers. Sometimes, how they combine is dictated by **convention**, like with the **order of operations**. Other times, though, properties about numbers exist simply due to how these operations work.

**Exercise #1:** Add the following numbers without using a calculator. Hint: Although **order of operations** tells us we should add from left to right, think about an easier way to sum these numbers. Show how you summed them.

\[3 + 9 + 4 + 2 + 7 + 1 + 6 + 8\]

Addition and multiplication have two very important properties with very technical names. The next exercise will review these properties.

**Exercise #2:** Fill in the missing blanks for each property.

(a) **Commutative Property of Addition:**

\[8 + 4 \text{ gives the same sum as } \underline{}_. \text{ Both sums equal } \underline{}_.\]

(b) **Commutative Property of Multiplication:**

\[6 \times 3 \text{ gives the same product as } \underline{}_. \text{ Both products equal } \underline{}_.\]

(c) **Associative Property of Addition:**

The sum \((3 + 5) + 9\) gives the same result as the sum \(\underline{}_.\)

Both sums are equal to \(\underline{}_.\)

(d) **Associative Property of Multiplication:**

The product \((2 \cdot 5) \cdot 7\) gives the same result as the product \(\underline{}_.\)

Both products are equal to \(\underline{}_.\)
The Commutative Property and Associative Property essentially give us permission to rewrite addition and multiplication problems in different orders than what are normally given.

**Exercise #3:** Give an example that shows that subtraction is not commutative.

Even though subtraction is not commutative, we should remember a very important fact about subtraction: it can always be made into the addition of opposites.

**Exercise #4:** Change the following expression involving addition and subtraction into one only involving addition and then use the commutative and associative properties to quickly determine the value of this expression. Through this, please review some properties of negative numbers.

\[7 - 3 + 8 - 2 - 6 + 1 - (-3)\]

We should be able to now extend the commutative and associative properties for numbers we know to numbers we don’t know (variables). One of the very nice ways to illustrate the usefulness of these properties is in combining two or more expressions.

**Exercise #5:** Please recall the following quickly:

(a) \(5x + 2x = \)  
(b) \(7x - 3x = \)  
(c) \(-8x + 2x = \)

**Exercise #6:** In the following exercise we show how two linear expressions are combined using various properties. List what the properties are:

\[(3x + 7) + (2x + 8) = 3x + 7 + 2x + 8\]
\[3x + 7 + 2x + 8 = 3x + 2x + 7 + 8\]
\[3x + 2x + 7 + 8 = (3x + 2x) + (7 + 8)\]
\[= 5x + 15\]

**Exercise #7:** Combine the expressions below. Replace subtraction by addition of opposites, if needed.

(a) \(4x + 6 - 2x - 9\)  
(b) \(-6x + 9 + 10x + 3\)  
(c) \(4y - 10 - 7y - 3\)
THE COMMUTATIVE AND ASSOCIATIVE PROPERTIES
COMMON CORE ALGEBRA I HOMEWORK

FLUENCY

1. Combine the expressions below. Replace subtraction by addition of opposites, if needed.

(a) \(7x + 3 + 6x + 11\)  
(b) \(12x + 10 + 3 + 8x\)  
(c) \(10y + 12 - 7y - 8 - 3y\)

(d) \(8x - 6 - 7x + 10\)  
(e) \(-6x + 9 + 4x - 9\)  
(f) \(-4x + 5 - 12 - 7x + 4 + 2x\)

(g) \(12x - 15 - 3 + 2x - 15x\)  
(h) \(-7x + 4 - 11 - 7x + 7 + 2x + 12x\)  
(i) \(-2x + 18 + 4x - 12 - 6\)

2. Use the associative property to rewrite the following. You do NOT need to simplify these.

(a) \(2 + (3 + 4) =\)
(b) \(5 \times (3 \times 7) =\)
(c) \(3x - (2x + 9x) =\)

3. Use the commutative property to rewrite the following. You do NOT need to simplify these.

(a) \(6 + 8 + 7\)
(b) \(12x + 8x - 3x\)
(c) \(-3y - 6y + 10y\)
APPLICATIONS

4. Sophia and Emily are twin sisters and best friends. They’re saving up for concert tickets and agreed to pay for the tickets together when they have enough money. They both created equations to see how fast they were making money and came up with the following expressions:

Sophia: $35w + 55 - 10w$

Emily: $28w + 75 - 5w + 12$

(a) Combine their expressions to see how much they are making together.

(b) Using the expressions see if they will have above $350 in four weeks. If not how much will they be short?

(c) If their friend Becky also wants to join and is making money according to the expression $50w + 25$, create a new expression for the total and see if they will have above $525 for the three of them after four weeks.

REASONING

5. List which of the associative and commutative properties are being used in each step.

$\left(9x - 3\right) + \left(10 - 5x\right) = 9x - 3 + 10 - 5x$

$9x - 3 + 10 - 5x = 9x - 5x - 3 + 10$

$9x - 5x - 3 + 10 = \left(9x - 5x\right) + \left(-3 + 10\right)$

$= 4x + 7$
In the last lesson we saw the important properties of addition and multiplication: the commutative and associative. The last of the three major properties combines addition and multiplication: the **distributed property**. The first exercise will illustrate the idea.

**Exercise #1:** Consider the product \( 4 \times 15 \).

(a) Evaluate using the standard algorithm. 
(b) Represent the equivalent product \( 4 \times (10 + 5) \) as repeated addition of 10 and 5. Find the product.

Exercise #1 shows the important property of being able to apply a multiplication to all parts of a sum. In symbolic form:

**The Distributive Property (of Multiplication Over Addition)**

If \( a \), \( b \), and \( c \) all represent real numbers then:  
\[ a(b + c) = a \cdot b + a \cdot c \]

**Exercise #2:** Evaluate each product by using the distributive property to make it easier. On (b), express 18 as a subtraction. Do not use a calculator.

(a) \( 7(23) \) 
(b) \( 9(18) \)

**Exercise #3:** The distributed property can be used twice in order to multiply two digit numbers. For example find the product \( (12)(28) \) by evaluating \( (10 + 2)(20 + 8) \). Show each step in your calculation. Do not use a calculator unless it is to check.
The distributive property can also be used on expressions that involve variables.

**Exercise #4:** Express the following products as binomial expressions. Show each step in your calculation.

(a) \(5(2x + 3)\)  
(b) \(-4(5x - 8)\)

(c) \(x(x + 4)\)  
(d) \(5x(2 - 7x)\)

One common mistake students make is not realizing that the distributive property works for division as well as multiplication. For division, the property would look in symbolic form like:

**THE DISTRIBUTIVE PROPERTY (OF DIVISION OVER ADDITION)**

If \(a\), \(b\), and \(c\) all represent real numbers then: \(\frac{b + c}{a} = \frac{b}{a} + \frac{c}{a}\)

**Exercise #5:** Express each of the following quotients as binomials in simplest form. Show your calculations. Some of your final answers will contain fractional coefficients.

(a) \(\frac{8x + 4}{2}\)  
(b) \(\frac{25x - 50}{5}\)

(c) \(\frac{2x - 16}{4}\)  
(d) \(\frac{-9x + 18}{12}\)
THE DISTRIBUTIVE PROPERTY
COMMON CORE ALGEBRA I HOMEWORK

FLUENCY

1. Using the equivalent expressions provided find the value of the product on the left by evaluating the expression on the right.

   (a) \(5(42) = 5(40 + 2)\)  
   (b) \(3(27) = 3(25 + 2)\)  
   (c) \(5(58) = 5(60 - 2)\)

2. Simplify the following expressions using the distributive property. Show your calculations.

   (a) \(2(4x + 2)\)  
   (b) \(4(3x - 1)\)  
   (c) \(3(7 - x)\)

   (d) \(\frac{36x + 21}{3}\)  
   (e) \(\frac{18 - 36x}{4}\)  
   (f) \(\frac{3(4x + 8)}{6}\)

APPLICATIONS

3. Using your knowledge of the distributive property, rewrite the following and evaluate without using your calculator. See Problem #1 if you need a hint how to do these.

   (a) \(6(38) = \)  
   (b) \(7(35) = \)
4. Nate noticed that when using the distributive property you multiply the term outside the parenthesis by **each** term inside. Using his realization see if you can multiply the following using the distributive property.

(a) \(3(246) = 3(200 + 40 + 6)\) 

(b) \(2(3269) = 2(3000 + 200 + 60 + 9)\)

(c) \(3(2x^2 + 4x + 6)\)

(d) \(2(5x^3 + 2x^2 + 6x + 9)\)

**REASONING**

5. In the lesson we saw that we can multiply 2 digit numbers by using the distributive property twice. Use this knowledge to multiply the following terms. Show the calculations that lead to your answers.

(a) \((22)(31)\)

(b) \((52)(11)\)

6. Which of the following is equivalent to \((2x + 2)(3x + 1)\)? It may help to use problem #5(a) as a reference.

(1) \(6x^2 + 2\)  

(2) \(5x^2 + 8x + 3\)

(3) \(6x^2 + 8x + 2\)

(4) \(16x^3\)
EQUIVALENT EXPRESSIONS
COMMON CORE ALGEBRA I

The idea of equivalent expressions, or equivalency, is extremely important. It is the basis of many if not most of our algebraic manipulations. The definition of equivalent expressions is given below.

**EQUIVALENT EXPRESSIONS**

Two (or more) algebraic expressions are equivalent if they have the same value for every value of the substitution variable (or variables). In other words, no matter what value you stick in for $x$ (or $y$ or $z$) the two expressions come out equal.

**Exercise #1:** Consider the three expressions below. By substituting in the values of $x$ given, determine which two expressions are equivalent. Show your calculations of the expressions’ values and circle your final answers.

<table>
<thead>
<tr>
<th></th>
<th>$5(x-3)$</th>
<th>$5x-3$</th>
<th>$5x-15$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x=7$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$x=0$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$x=1$</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Exercise #2:** Which property, the commutative, associative, or distributive, justifies the equivalency of the two expressions you determined to be equivalent above?

**Exercise #3:** Which of the following expressions is equivalent to $5(2x+1)-4$? Show your work to justify your response. Test at least one value of $x$ to check your answer.

(1) $10x-3$  
(2) $7x-3$  
(3) $10x+1$  
(4) $7x+1$
**Exercise #4:** Which of the following expressions is equivalent to $\frac{4(3x+1) - 2}{2} - 5$? Again, show your work by thinking carefully about order of operations and the properties we have learned about. Finally, check your answer by substituting a value of $x$. Show this check.

(1) $4x - 3$  
(2) $4x + 1$  
(3) $6x + 3$  
(4) $6x - 4$

The last exercise is an example of an expression with a fair number of operations within it. Sometimes, it is just as important to recognize more simple equivalencies.

**Exercise #5:** Which of the following expressions is equivalent to $10x + 15$? Explain how you made your choice in the space provided.

(1) $2(8x + 13)$  
(2) $5(2x + 3)$  
(3) $5(5x + 3)$  
(4) $10(x + 5)$

The last problem is an example of what is known as **factoring**.

---

**Factoring Expressions**

**Factoring** is the process of writing an **equivalent expression** as purely the product of other expressions.

Factoring will be one of the most important skills that we want to reach **fluency** with, but for now we will do some fairly easy factoring by simply applying the **distributive property** in “reverse” if you will.

**Exercise #6:** Factor each of the following expressions by writing an equivalent expression that is in the form of a product. Check your work by using the distributive property.

(a) $6x + 21$  
(b) $-2x + 10$  
(c) $14x + 14$
**EQUIVALENT EXPRESSIONS**

**COMMON CORE ALGEBRA I HOMEWORK**

**FLUENCY**

1. Use the Associative, Commutative and Distributive properties to write the expression given as an equivalent expression in simplest form.

   (a) \(2x + 8 + 3x - 3\) 
   (b) \(3x + (5x + 2x)\) 
   (c) \((3x - 4) + (2x + 1)\)

   (d) \(6(2 - 3x) + 1\) 
   (e) \(x + 4 - 2\left(\frac{1}{2}x + 3\right)\) 
   (f) \(3(x + 2) - 2(x + 1)\)

   (g) \(\frac{12x + 18}{6}\) 
   (h) \(\frac{2(5x + 3) - 4}{2} + 1\) 
   (i) \(\frac{1}{2}(4x + 8) - 8\)

2. Factor each of the following by using the distributive property.

   (a) \(14x + 21\) 
   (b) \(6 - 3x\) 
   (c) \((2x + 4) + (3x - 14)\)
APPLICATIONS

3. Four friends have an assortment of Snack bars that cost $S$ dollars each, Munch bars that cost $M$ dollars each and Chewies that cost $C$ dollars each that they sell to raise money for a trip they are taking. They decide to split the money from the sales evenly between the four friends. They create an expression to make sure everyone gets the same amount. The amount each friend receives is given by the complicated expression

$$\frac{(5C + 5S) + (2M + 4S) + (10C + M) + (C + 3S + M)}{4}$$

(a) Write an equivalent expression that simplifies the amount that each friend will earn in terms of the unit costs $S$, $M$, and $C$.

(b) If Snack bars cost $3$ each, Munch bars cost $5$ each and Chewies cost $4.50$ each, then how much does each friend earn?

REASONING

4. Taylor is factoring the following expression but notices she got the wrong answer when checking her work. Identify what she did wrong and show her the appropriate way to factor.

Taylor’s work:  Your work:

$12x + 3 = 3(4x)$

Taylor’s Check:  Your check:

$3(4x) = 12x$ #

5. State which property (associative, commutative, or distributive) was used to get from one equivalent expression to the next.

- $2(3x + 5) + 4(2x - 1)$
  $= -6x - 10 + 4(2x - 1)$
  $= -6x - 10 + 8x - 4$
  $= -6x + 8x - 10 - 4$
  $= (-6x + 8x) + (-10 - 4)$
  $= (-6 + 8)x - 1(10 + 4)$
  $= 2x - 14$
SEEING STRUCTURE IN EXPRESSIONS
COMMON CORE ALGEBRA I

Many times the techniques of algebra can seem like mindless moving of symbols from here to there without any obvious purpose. In the Common Core, we seek to challenge students to do mindful manipulations. In other words, always have a reason for the manipulation you are doing.

MINDFUL MANIPULATION
A mindful manipulation will be an algebraic technique applied with a purpose in mind, even if we are unsure if the manipulation will results in success. In this sense, we want to give ourselves permission to do manipulations even if they fail to reach our purpose.

The exercises in this lesson are about problem solving and using the properties we have learned about in mindful ways to try to solve problems that are puzzle like in nature. But, we will start with a problem that illustrates the idea.

Exercise #1: Consider the expressions $2x + 1$ and $6x + 3$.

(a) Find the value of both expressions when $x = 2$.

(b) What is the ratio of the larger outcome to the smaller? Why did the ratio turn out this way? What property can you use to justify this?

O.k. Now for some problems that are a bit more challenging. You are going to be asked for the value of an expression without knowing the value of $x$. Let’s do a warm up.

Exercise #2: The expression $3x + 2$ is equal to 7 for some value of $x$ (don’t solve for it!). Determine the values of each of the following expressions for the same value of $x$. Show your reasoning.

(a) $6x + 4$  
(b) $3x + 5$
The last exercise forces us to do mindful manipulations because we have to think about how expressions relate to each other and how to write equivalent expressions. Let’s do some more of these types of puzzles.

**Exercise #3:** The expression $2x + 5$ has a value of 10 for some value of $x$ (don’t solve for it). Do mindful manipulations on each of the following to find the values of these expressions for this same value of $x$.

(a) $4x + 10$  
(b) $2x + 20$  
(c) $2x + 1$

(d) $-2x - 5$  
(e) $10x + 25$  
(f) $2x - 5$

(g) Challenge: Find the value of $6x + 20$. Hint: just try to come close and see what else you have to do in terms of adding or subtracting.

**Exercise #4:** If the expression $3x - 4$ has a value of $-3$ for some value of $x$, then what is the value of the expression $(3x - 4)^2 + 6x - 8$ for the same value of $x$? Show the calculations that lead to your choice below.

(1) 11  
(2) $-15$  
(3) 3  
(4) $-6$
SEEING STRUCTURE
COMMON CORE ALGEBRA I HOMEWORK

FLUENCY

1. Get a warm-up with the following. Evaluate each expression for the given value of \(x\). Do these without the aid of a calculator to practice your mental arithmetic.

   (a) \(3x - 8\) for \(x = 5\)
   
   (b) \(5(x + 7) - 1\) for \(x = -3\)
   
   (c) \(\frac{x - 8}{4} + 5\) for \(x = 4\)

2. If the expression \(x - 3\) has a value of \(-5\), then which of the following represents the value of \(3x - 9\)? Explain how you arrived at your choice.

   (1) \(-2\)
   
   (2) \(-9\)
   
   (3) \(-15\)
   
   (4) \(-42\)

3. The expression \(2x + 6\) is equal to 9 for some value of \(x\). Without finding the value of \(x\), determine the values for each of the following expressions. Show how you arrived at each answer.

   (a) \(4x + 12\)
   
   (b) \(2x + 9\)
   
   (c) \(x + 3\)
   
   (d) \(-6x - 18\)
   
   (e) \(2x + 1\)
   
   (f) \(10x + 32\)

4. The expression \(x - 2\) has a value of \(-5\) for some value of \(x\). For the same value of \(x\), what is the value of the expression \((x - 2)^2 + 5x - 10\)? Show your reasoning for this problem in the space provided.

   (1) 0
   
   (2) 15
   
   (3) 14
   
   (4) \(-10\)
APPLICATIONS

5. The number of feet that Jennifer can run in a given time period $t$ is given by the expression $8t + 2$. Her friend Erika can run a distance given by the expression $4t + 3$. Erika claims that she can only run half of what Jennifer can plus an additional 2 feet. Is she correct?

(a) Let’s build up some evidence by playing around with various values of $t$. Fill out the following chart for both Jennifer and Erika’s distances given the value of $t$.

<table>
<thead>
<tr>
<th>Time, $t$</th>
<th>Jennifer’s Distance $8t + 2$</th>
<th>Erika’s Distance $4t + 3$</th>
<th>Is Erika Correct?</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(b) The table provides good numerical evidence that what Erika says is true. Show by using mindful manipulations of the expression $8t + 2$ that Erika’s distance is always 2 feet more than half of Jennifer’s.

REASONING

We can use the same sort of reasoning to help solve equations. We haven’t done much of that yet but try your best to think about these problems in the context of solving an equation.

6. Say I knew that the solution to the equation $2x - 7 = 9$ was $x = 8$. How could I use that to help me to solve the equation $2(x + 2) - 7 = 9$?

(a) Either $x = 6$ or $x = 10$ is a solution to our new equation: $2(x + 2) - 7 = 9$. Check to see which is a solution by substituting them into the left hand expression and seeing if it is equal to 9.

(b) Explain in your own words why the correct answer was the one you found in (b). In other words, look at the structure of both equations. Think about what is the same and what is different.
We’ve used exponents a little so far, but they will become much more important to us as our studies in algebra progress. So, in the next few lessons we are going to work with some basic exponents. Recall that an exponent is a way to indicate repeated multiplication by the same number.

**Exercise #1:** Write out what each of the following exponents means as an extended product and find its value.

(a) $2^4$  
(b) $3^2$  
(c) $5^3$

Of course, just as with numbers, variables can also be raised to exponents (other than 1).

**Exercise #2:** Write out what each of the following terms involving exponents means as an extended product. Consider carefully your order of operations and remember that exponents come before multiplication.

(a) $x^3$  
(b) $x^2y^4$  
(c) $(2x)^2$

(d) $4x^4y^3$  
(e) $(9x^2)^3$  
(f) $(-4x^3)^2$

One of the nice aspects of exponents is that they follow very predictable patterns, often known as **exponent rules** (and they DO RULE!). Let’s figure out the simplest one in the next exercise.

**Exercise #3:** Write out each of the following products and then express them in the form $x^n$.

(a) $x^2x^3$  
(b) $x^5x^2$  
(c) $x^4x^4$
**Exercise #4:** So, what’s the pattern? Can you give a generic rule for what happens when we multiply two terms that have the same base?

**Exponent Rule #1:**  
\[ x^a \cdot x^b = \]

This exponent rule allows us to multiply larger powers of variables without actually having to write out the products. Make sure to internalize this rule. In other words, think about it until you are absolutely sure you understand why it works. Eventually, we will extend exponent rules to weird situations with all sorts of exponents.

**Exercise #5:** Quickly write each of the following products as a variable raised to a single power.

(a) \( x^4x^9 \)  
(b) \( x^2x^3x^4 \)  
(c) \( y^2y^6 \)

Often you will need to be able to multiply more complicated terms and write them in as convenient (simplest) form as possible. The next exercise walks us how to do this and the laws and properties needed.

**Exercise #6:** The steps to simplifying the product: \( 5x^3 \cdot 2x^7 \) to simplest terms are shown below. Write in what justifies each step.

Step 1: \( 5x^3 \cdot 2x^7 = 5 \cdot 2 \cdot x^3 \cdot x^7 \)  
Justification: ______________________________________

Step 2: \( 5 \cdot 2 \cdot x^3 \cdot x^7 = (5 \cdot 2) \cdot (x^3 \cdot x^7) \)  
Justification: ______________________________________

Step 3: \( (5 \cdot 2) \cdot (x^3 \cdot x^7) = 10x^{10} \)  
Justification: ______________________________________

Each step in an algebraic manipulation can ultimately be justified using a property that was established about numbers (like the commutative) or a pattern, like Exponent Rule #1 above. But, we also need to become fluent in these manipulations. The next exercise gives you some opportunity to do so.

**Exercise #7:** Rewrite each of the following as equivalent expressions in simplest exponential form.

(a) \( 2x^7 \cdot 8x^5 \)  
(b) \( (-4x^3)(2x^2) \)  
(c) \( (-6x^3)^2 \)
**FLUENCY**

1. Rewrite each of the following terms as an extended product. Consider carefully your order of operations and remember that exponents come before multiplication. You do not need to simplify the products.

   (a) \(4^3\)  
   (b) \(3^2 \cdot 3^3\)  
   (c) \(2^3)^4\)  
   (d) \(x^3 y^4\)  
   (e) \(8x^2 y^5\)  
   (f) \((9x^2)^2\)

2. Write out each of the following products and then express them in simplest exponential form.

   (a) \(x^4 x^7\)  
   (b) \(y^3 y^6\)  
   (c) \(x^3 y^2 x^5 y^2\)

3. Rewrite each of the following as equivalent expressions in simplest exponential form. There is one that cannot be simplified. Identify it.

   (a) \(4x^3 \cdot 7x^6\)  
   (b) \(x^5 y^3 x^2\)  
   (c) \((-x^2)(3x^{10})\)

   (d) \(x^2 y^3 z^3\)  
   (e) \((4x)^3\)  
   (f) \((-3x^2)^2\)
APPLICATIONS

4. One of the most common uses of exponents is when dealing with **scientific notation**. Recall that $3.2 \times 10^4$ is written in scientific notation where 10 is being raised to the 4th power. If $3.2 \times 10^4$ is the length of a park in meters and $2.5 \times 10^6$ is the width in meters, what is the area of the park if it is in the shape of a rectangle? It may help to write the terms out as an extended product and then regroup them.

$$\text{Area} = \text{Length} \times \text{Width} = (3.2 \times 10^4)(2.5 \times 10^6) =$$

REASONING

5. The steps to simplifying the product $\left(2x^3\right)^3$ to simplest terms are shown below. Write in what justifies each step.

Step 1: $(2x^3)^3 = 2x^3 \cdot 2x^3 \cdot 2x^3$
Justification: __________________________

Step 2: $2x^3 \cdot 2x^3 \cdot 2x^3 = 2 \cdot 2 \cdot 2 \cdot x^3 \cdot x^3 \cdot x^3$
Justification: __________________________

Step 3: $(2 \cdot 2 \cdot 2) \cdot (x^3 \cdot x^3 \cdot x^3) = 8x^9$
Justification: __________________________

6. So far we have come up with an exponent rule for the multiplying two monomials with like bases. We saw this to be $x^a \cdot x^b = x^{a+b}$. We can also find a rule for simplifying the expression $(x^a)^b$. Try the following questions and see if you can find the pattern that helps simplify this type of expression.

(a) Rewrite the following terms as extended products and then express them in the form $2^n$ or $x^a$.

(i) $\left(2^2\right)^4$
(ii) $\left(x^3\right)^4$

(b) Looking back at part (a) see if you can see a connection between your answer and the question. Make a general rule for all terms in the form of $(x^a)^b$

$$(x^a)^b = \text{KNOW THIS RULE!!!}$$
We should now have a better ability to work with exponents. In this lesson we will continue to explore expressions that are equivalent but look different. We will be primarily sticking with linear expressions (those where \(x\) is only raised to the first power) and quadratic expressions (where \(x\) is raised to the second power).

Recall that two expressions are equivalent if they return equal values when values are substituted into them.

**Exercise #1:** Consider the product \((x-2)(x+5)\). It is equivalent to one of the expressions below. Determine which by substituting in two values of \(x\) to check.

<table>
<thead>
<tr>
<th></th>
<th>((x-2)(x+5))</th>
<th>(x^2 -10)</th>
<th>(x^2 +3x -10)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(x=3)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(x=5)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The last exercise is pretty interesting. It would seem that if you were just mindlessly manipulating the product of the two binomials, then you would likely think two expressions were equivalent, when they are not. Let’s find out in the next exercise how to multiply out two simple binomials using a variety of properties.

**Exercise #2:** The steps in finding the product of \((x+3)(x+5)\) are shown below. Write down the justification for each step.

Step 1: \((x+3)(x+5)= (x+3) \cdot x + (x+3) \cdot 5\)  
Justification: ______________________________

Step 2: \((x+3) \cdot x + (x+3) \cdot 5 = x \cdot x + 3 \cdot x + x \cdot 5 + 3 \cdot 5\)  
Justification: ______________________________

Step 3: \(x \cdot x + 3 \cdot x + x \cdot 5 + 3 \cdot 5 = x \cdot x + x(3+5) + 3 \cdot 5\)  
Justification: ______________________________

\[= x^2 + 8x + 15\]
Wow! That’s a lot of justification. The process of multiplying binomials is very important in algebra, so it’s another skill we would like to get fluent with. Get some practice in the next exercise. Keep in mind that you are simply doing the distributive property twice.

**Exercise #3:** Write out each of the following as equivalent trinomials (an expression involving three terms).

(a) $(x + 6)(x + 3)$  
(b) $(x - 4)(x + 6)$  
(c) $(x - 3)(x - 3)$

(d) $(2x + 3)(3x + 1)$  
(e) $(3x - 4)(3x + 2)$  
(f) $(4x - 1)(x - 7)$

**Exercise #4:** Jeremy has noticed a pattern that he thinks is always true. If he picks any number and finds the product of one number larger and one number smaller than it, the result is always one less than the square of his number.

(a) Test some numbers out and see if Jeremy’s pattern holds.

(b) Give an algebraic explanation that shows that Jeremy’s pattern will work for any number. Use let statements to clearly define your variables.

**Exercise #5:** Which of the following expressions is equivalent to the product $(x - 2)(x - 4)$? Show the calculations that you use to find your choice and test using a value of $x$.

(1) $x^2 + 8$  
(2) $x^2 - 6x - 8$  
(3) $x^2 - 6x + 8$  
(4) $x^2 - 8$
MORE COMPLEX EQUIVALENCY
COMMON CORE ALGEBRA I HOMEWORK

FLUENCY

1. Rewrite each expression as a simpler, equivalent expression by first using the Distributive Property and then combining terms.

(a) \( x(x - 2) \)

(b) \( x(x + 6) + 3(x + 6) \)

(c) \( (x + 3)(x + 6) \)

(d) \( 4x(2x + 3) \)

(e) \( (3x - 4)(3x + 2) \)

(f) \( (x + 3)(x - 3) \)

(g) \( (3x + 4)(2x - 1) \)

(h) \( (x - 3)(x - 3) \)

(i) \( (x - 2)^2 \)

2. Which of the following expressions is equivalent to \( (x + 7)^2 \)? Test with a value of \( x \). Show your test.

(1) \( x^2 + 49 \)

(3) \( (x + 7)(x + 7) \)

(2) \( (x - 7)(x + 7) \)

(4) \( (7x)(7x) \)

3. Continuing with the expression \( (x + 7)^2 \), do the following.

(a) By using the Distributive Property twice, show that this expression is equivalent to \( x^2 + 14x + 49 \).

(b) Test the equivalency by finding the value of \( (x + 7)^2 \) and \( x^2 + 14x + 49 \) when \( x = 3 \).

\[ (x + 7)^2 \quad x^2 + 14x + 49 \]
APPLICATION

4. When reading some schematics of a rectangular garden you see the binomial $x + 8$ feet represents the length and the binomial $x - 1$ feet represents the width. Write an expression that represents the total area of the garden in the form $x^2 + bx + c$ by using the distributive property.

Recall that $\text{Area} = \text{Length} \times \text{Width}$

(b) Test to make sure that your expression from above is equivalent to $(x - 1)(x + 8)$ using the following values of $x$. Show your tests for equivalency.

$x = 3$ $(x - 1)(x + 8)$ Your Expression:

$x = 10$ $(x - 1)(x + 8)$ Your Expression:

REASONING

5. Mariah thinks that the following rule should always hold true. Should it? Find evidence for or against the following equivalency rule by substituting various values in for $a$ and $b$.

$\left(a + b\right)^2 = a^2 + b^2$

6. Using your understanding of the distributive property, write an equivalent expression of $\left(a + b\right)^2$ in terms of $a$ and $b$. Hint: if you’re having trouble, try referencing problem #2 and #3.
MORE STRUCTURE WORK
COMMON CORE ALGEBRA I

The more you are able to see structure in the various expressions that you deal with, the easier it will be to manipulate complex expressions. We will work with structure throughout the course. We can now look at some larger structural issues that include equivalency.

Exercise #1: Consider the somewhat complex expression \( x(x + 4) + 2(x + 4) \).

(a) Write an equivalent trinomial expression. Test the equivalency with a value of \( x = 1 \). Show the test.

(b) Write an equivalent expression that is in the form of a product of two binomials. Also test the equivalency with \( x = 1 \).

Which type of equivalent expression we might need would depend on the context of what we were trying to do with the math. For now, we want to get practice with writing various expressions in an equivalent form, and being able to test that equivalency.

Exercise #2: Consider the expression \( (x + 4)(x - 5) + (x + 4)(x - 2) \). Write an equivalent expression that is in the form of the product of two binomials. Test the equivalency with a value of \( x \). Show your test.

Exercise #3: Which of the following is equivalent to the expression \( (x - 3)(2x + 7)-(x - 3)(x - 4) \)? Show the manipulations that lead to your choice.

(1) \((x - 3)(x + 3)\)  
(2) \((x - 3)(x + 11)\)  
(3) \((x - 6)(x + 10)\)  
(4) \((x - 6)(x - 4)\)
Strangely enough, this type of manipulation, where there is a common binomial multiplying two other terms, is frequent enough that it is also a good skill to become **fluent** in. Get some additional practice in the next exercise. Be careful when subtraction is involved (see the last exercise!).

**Exercise #4**: Rewrite each of the following expressions as an equivalent product of two binomials.

(a) \( x(x+5)+7(x+5) \)  
(b) \( 3x(x-2)-4(x-2) \)  
(c) \( -2x(x+4)+x+4 \)

(d) \( (x-6)(x+3)+(x+9)(x+3) \)  
(e) \( (2x+1)(x-4)-(x+6)(x-4) \)

Remember, that we want to always look for **mindful manipulations** in order to help us solve our problems. Sometimes we won’t know whether those manipulations will pay dividends, but as long as we know we are making manipulations that retain **equivalency** then they are worth a try.

**Exercise #5**: The binomial \( 4n+1 \) is equal to 7 for some value of \( n \). What is the value of the expression shown below for the same value of \( n \). Do not solve for \( n \) in this problem. Use mindful manipulations and look for structure to help solve this problem.

\[
(3n+1)(4n+1)+(n+2)(4n+1)
\]
**Fluency**

1. Rewrite each of the following expressions as an equivalent product of two binomials.

   (a) \( x(x+2)+3(x+2) \)  
   (b) \( x(x-1)-4(x-1) \)  
   (c) \( 2x(x+4)+3(x+4) \)  
   (d) \(-2x(x+12)+3(x+12)\)  
   (e) \(3(x-5)+3x(x-5)\)  
   (f) \(-4x(x+3)+3x^2(x+3)\)
   
   (g) \(2x-7)(x+2)+(3x+7)(x+2)\)  
   (h) \(2x+5)(x-4)-(x-4)(5x+2)\) 

2. Which of the following choices is equivalent to the expression \((x-2)(6-4x)+(5x+4)(x-2)\)? Show the calculations that lead to your choice and check using a value of \(x\).

   (1) \((x-2)(x+2)\)  
   (2) \((x-2)(9x+10)\)  
   (3) \((x-2)(x+10)\)  
   (4) \((x-2)(10-9x)\)

3. If \(x+2\) has a value of 5, then which of the following is the value of \(x(x+2)+3(x+2)\)? Show the work that leads to your answer.

   (1) 30  
   (2) 25  
   (3) 15  
   (4) 10
APPLICATIONS

4. When figuring out the amount of mulch would be needed for Alex’s back yard, he created an equation that approximates the number of bags, \( B \), he’ll use. If his equation is \( B = 4(2x + 7) + 3(2x + 7) \) and \( (2x + 7) \) is equal to 2, how many bags will he need? Show your mindful manipulations.

5. Alex’s friend Pablo comes up with an exact equation to find out how many bags he needs. Use his equation to find out how many bags will actually be needed if \( B = x(2x + 7) + 3(2x + 7) + (x + 4)(2x + 7) \), where the quantity \((2x + 7)\) equals 4. Show how you arrive at your answer.

REASONING

6. In most of the previous examples there were only two terms. Extend your work with using the Distributive Law “backwards” and write the following as a product of binomials.

   (a) \( x(x + 2) + 3(x + 2) + 4x(x + 2) \)  
   (b) \( 2x(x - 5) + 3(x - 5) + (x - 1)(x - 5) \)

7. Write \( x(x - 2) + 3(2 - x) \) as a product of binomials. Hint: you may want to manipulate \((2 - x)\) first. Check to see if you have written an equivalent expression by testing \( x = 5 \).
TRANSLATING ENGLISH TO ALGEBRA
COMMON CORE ALGEBRA I

There will be many instances when we have to translate phrases from English into mathematical expressions. This is a skill that takes a lot of practice and time to get good at. In this lesson we will begin to build this fluency.

**Exercise #1:** It is important to be able to recognize addition and subtraction in phrases. First, let’s begin with some numerical work and then transition to expressions that contain variables.

(a) Write a calculation and a result that represents a number that is 5 greater than 3.

(b) Write a calculation and a result that represents a number that is 2 less than 9.

(c) Write a calculation and a result that represents the sum of $-3$ and 8.

(d) Write a calculation and a result that represents the difference of 20 and 12.

(e) If $x$ represents a number, write an expression that represents a number 10 greater than $x$.

(f) If $n$ represents a number, write an expression that represents a number that is 5 less than $n$.

(g) If $y$ represents a number, write an expression that represents the sum of $y$ and a number one greater than $y$.

(h) If $n$ represents a number, write an expression that represents the difference between a number one larger than $n$ and one smaller than $n$. Be careful.

We also need to be able to translate multiplication and division. Multiplication is typically easier to spot and translate. Let’s get some practice.

**Exercise #2:** Translate each verbal statement into an expression and evaluate the expression if it is numerical.

(a) Write an expression for a number that is five times greater than 2.

(b) If $n$ represents a number, then write an expression for a number that is twice $n$.

(c) Write an expression for the quotient (or ratio) of 12 and 3.

(d) If $x$ represents a number, write an expression for the ratio of $x$ to 5.
Now we want to be able to put operations together to create more complex expressions. These can be tricky. It is always important to read them carefully, think about your order of operations, and check with a real number.

**Exercise #3:** Translate each of the following statements into an algebraic expression.

(a) If $x$ represents a number, then write an expression for a number that is three more than twice the value of $x$.

(b) If $n$ represents a number, then write an expression for two less than one fourth of $n$.

(c) If $s$ represents Sally’s age and her father is 4 years less than five times her age, then write an expression for her father’s age in terms of the variable $s$.

(d) If $x$ represents a number, then write an expression for three times the sum of $x$ and 10.

(e) If $n$ represents a number, then write an expression for 7 less than four times the difference of $n$ and 5.

(f) If $x$ represents a number, then write an expression for the ratio of 3 less than $x$ to 2 more than $x$.

(g) If $x$ represents a number, then write an expression for the sum of twice $x$ with twice a number one larger than $x$.

(h) If $n$ represents a number, then write an expression for the quotient of twice $n$ with three less than $n$.

(i) If $y$ represents a number, then write an expression for three-quarters of the difference of $y$ and 8.

(j) If $x$ represents a number, then write an expression for one half the sum of $x$ and 4.

**Exercise #4:** Neat patterns can occur repeatedly when you play around with numbers. A fairly easy one occurs when you add a number to one less and one more than the number. Do this for a few numbers, $x$, and record the results. Then, prove a general pattern by writing an expression for the sum of a number with a number one less and a number one more than it.

<table>
<thead>
<tr>
<th>CALCULATIONS:</th>
<th>$x$</th>
<th>sum</th>
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<table>
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<tr>
<th>ALGEBRAIC EXPRESSION:</th>
<th>$x$</th>
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</table>
TRANSLATING ENGLISH TO ALGEBRA
COMMON CORE ALGEBRA I HOMEWORK

FLUENCY

1. Translate each of the following statements into an algebraic expression.

(a) If $x$ represents a number, then write an expression for a number that is three more than the number.

(b) If $x$ represents a number, then write an expression for a number that is eight less than twice the value of $x$.

(c) If $x$ represents a number, then write an expression for a number that is three more than one third the value of $x$.

(d) If $n$ represents a number, then write an expression for two less than one fourth of $n$.

(e) If $g$ represents Greg’s age and his daughter is 4 years less than one half his age, then write an expression for his daughter’s age in terms of the variable $g$.

(f) If $y$ represents a number, then write an expression for negative two times the sum of $y$ and 7.

(g) If $n$ represents a number, then write an expression for three times the difference of the number and six increased by four times the number.

(h) If $k$ represents a number, then write an expression for the ratio of 3 less than $k$ to 2 more than $k$.

(i) If $n$ represents a number, then write an expression for the difference of three times the number after it was increased by 3 and twice that number.

(j) If $h$ represents a number, then write an expression for the quotient of twice $h$ and 10 more than $h$.

(k) If $x$ represents a number, then write an expression for 7 more then one half the number.
APPLICATIONS

2. The Miller family made mathematical statements out of their ages as follows. Tom is four less than twice Gary’s age. Rebecca is the youngest and she is two less than half Gary’s age after it was increased by three. Sam’s age is the ratio of seven more than Gary’s age to eight less than Gary’s age.

(a) Translate each of the Miller family members ages into algebraic expressions in terms of Gary’s age, \(g\).

Tom’s Age: \(_{\quad}\) Rebecca’s Age: \(_{\quad}\) Sam’s Age: \(_{\quad}\)

(b) If Gary is 11 years old how old are each of the family members?

(c) Using Gary’s age come up with an expression that represents your age in terms of \(g\). Be creative! For example, if Mr. Weiler is 43 years old, then his age would be \(4g - 1\).

REASONING

Our future work in this course will necessitate that we work with what are known as consecutive integers. Integers are the set of positive and negative whole numbers (as well as zero).

The Integers: \(\{... -4, -3, -2, -1, 0, 1, 2, 3, 4...\}\)

Consecutive integers are lists of integers that increase by one unit between each.

3. Fill in the pattern with consecutive integers:

(a) 2, 3, _______, 5, _______, _______, 8 \hspace{1cm} (b) \(n\), \(n+1\), _______, \(n+3\), _______, _______, _________

4. We can also talk about consecutive even integers and consecutive odd integers. Fill in the patterns.

(a) 5, 7, 9, _______, 13, _______, _______, \hspace{1cm} (b) \(-10\), \(-8\), _______, _______, \(-2\), _______, _________

5. Regardless of whether we have consecutive even integers or consecutive odd integers, to get from one to another you add what number? If \(n\) represents the first in a list of consecutive even (or odd) integers, write out the next three terms.

What do we add? \(n\), _______, _______, _________
ALGEBRAIC PUZZLES
COMMON CORE ALGEBRA I

The new Common Core Algebra I curriculum challenges us to make sense out of mathematical patterns by utilizing the tools of algebra. Today we will explore patterns and see if we can justify them by manipulating algebraic expressions. Be mindful of your manipulations.

Exercise #1: Choose any number. Create the sum of two more than three times the number with two less than two times the number. What patterns is true of the result?

(a) Let’s explore the pattern with numbers we know before working with algebraic expressions. Even using numbers, the English is a bit tricky to decipher. Let’s do it together using a single number.

Let the number we choose be 3. Show the calculation as described in the problem.

(b) O.k. Now, let’s set up a table using various values to see if we can find a pattern.

<table>
<thead>
<tr>
<th>Number</th>
<th>Calculation</th>
<th>Results</th>
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</tbody>
</table>

Pattern:

(c) Now, let’s prove that the result that you see in the table will always be true. Let the number now be called \(x\), write an expression that translates the verbal description given in the problem for our calculation.

Exercise #2 (Fluency Problem): If \(n\) represents a number, which of the following expressions represents the sum of one more than twice the number and three less than five times the number?

(1) \(7n - 2\)  
(2) \(3n - 7\)  
(3) \(2n + 7\)  
(4) \(5n + 4\)
More interesting patterns can occur when products (multiplication) are brought in. Let’s try one that involves a product.

**Exercise #3:** In this problem we will explore a calculation of the difference between the product of a number and a number five larger than it and the product of the number and a number five less than it. Will this reveal a pattern like the last one?

(a) Like before, let’s explore the pattern with numbers we know before working with algebraic expressions.

Let the number we choose be 10. Show the calculation as described in the problem.

(b) O.k. Now, let’s set up a table using various values to see if we can find a pattern.

<table>
<thead>
<tr>
<th>Number</th>
<th>Calculation</th>
<th>Results</th>
</tr>
</thead>
<tbody>
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</tbody>
</table>

**Pattern:**

(c) Now, let’s prove that the result that you see in the table will always be true. Let the number now be called $x$, write an expression that translates the verbal description given in the problem for our calculation.
**ALGEBRAIC PUZZLES**

**COMMON CORE ALGEBRA I HOMEWORK**

**FLUENCY**

1. Use the table below to find a pattern for the *sum of 4 times a number* and *twice the sum of the same number and 3*.

<table>
<thead>
<tr>
<th>Number</th>
<th>Calculation</th>
<th>Results</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
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<tr>
<td>5</td>
<td></td>
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</tbody>
</table>

**Pattern:**

Now, let’s prove that the result that you see in the table will always be true. Let the number now be called $x$, write an expression that translates the verbal description given in the problem for our calculation.

2. Use the table below to find a pattern of the *difference of one more than six times a number* and *four more than three times the same number*.

<table>
<thead>
<tr>
<th>Number</th>
<th>Calculation</th>
<th>Results</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
<td></td>
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<tr>
<td>7</td>
<td></td>
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</tbody>
</table>

Try to determine the pattern by allowing the number to be called $x$. Write an expression that translates the text in italics above and then mindfully manipulate to see the pattern. How would you describe the pattern to a younger student?
3. If \( t \) represents a number, which of the following represents the product of 2 more than 5 times a number and 4 less than 3 times a number? Be sure to test a value of \( t \).

(1) \( 15t^2 + 26t - 8 \)  
(2) \( 15t^2 - 14t - 8 \)  
(3) \( 15t^2 - 8 \)  
(4) \( 15t^2 - 26t - 8 \)

APPLICATIONS

4. The length of a rectangle is two less than three times a number \( x \) and the width is five more than that same number.

(a) Draw a diagram that represents the rectangle. Be sure to label the sides in terms of the unknown \( x \).

(b) Using your diagram find what the perimeter of the rectangle is in terms of \( x \). Write your answer as a simplified binomial.

(c) What is the area of the rectangle in terms of \( x \)? Write your answer as a trinomial. Remember the formula for area of a rectangle is \( A = l \cdot w \).

REASONING

5. When finding a pattern for the sum of two times a number \( n \) and four less than three times the same number, Cole does the following calculation and decides that the pattern is \( 3n \). Show why he is wrong and find the correct pattern. Be sure to explain.

Cole’s work

\[
\text{Trial} \\
\begin{align*}
 n &= 2 \\
 2(2) + 3(2) - 4 &= 6
\end{align*}
\]
UNIT #2

LINEAR EXPRESSIONS, EQUATIONS, AND INEQUALITIES

Lesson #1 – Equations and Their Solutions
Lesson #2 – Seeing Structure to Solve Equations
Lesson #3 – A Linear Equation Solving Review
Lesson #4 – Justifying Steps in Solving an Equation
Lesson #5 – Linear Word Problems
Lesson #6 – More Linear Equations and Consecutive Integer Games
Lesson #7 – Solving Linear Equations with Unspecified Constants
Lesson #8 – Inequalities
Lesson #9 – Solving Linear Inequalities
Lesson #10 – Compound Inequalities
Lesson #11 – More Work with Compound Inequalities
Lesson #12 – Interval Notation
Lesson #13 – Modeling with Inequalities
A LOT of time is spent in Algebra learning how to solve equations and then solving them for various purposes. So, it goes without saying that we really need to understand what it means for something to “solve” an equation. First, let’s make sure we understand what an equation is:

**EQUATION DEFINITION**
An equation is simply a statement about the equality of two expressions. In other words, anything that takes this form:

\[ \text{Expression #1} = \text{Expression #2} \]

**Exercise #1:** Which of the following is not an equation?

1. \(3 + 1 = 4 + 0\)
2. \(x^2 - 2x = 8\)
3. \(2(4x + 1)\)
4. \(1 + 3 = 6\)

Equations can be either true, like (1) above, or false, like (4) above, depending on whether the two expressions are equal (true) or not equal (false).

**Exercise #2:** Consider the equation \(2x - 8 = 10 - x\).

(a) Why can’t you determine whether this equation is true or false?

(b) If \(x = 5\), will the equation be true? How can you tell?

(c) Show that \(x = 6\) makes the equation true. Remember to think very carefully always about your order of operations.

**SOLUTIONS TO EQUATIONS**
A value for a variable is called a solution to the equation if, when substituted into both expressions, results in the equation being true.
This concept of the solution to an equation is **amazingly important**. It implies that you can always know when you have solved an equation correctly. As long as you can check the truth of the equation with arithmetic, then you will know if your value (of $x$ often) is correct.

**Exercise #3:** Determine whether each of the following values for the given variable is a solution to the given equation. Show the calculations that lead to your final conclusions.

(a) $2x + 3 = 17$ and $x = 7$  
(b) $\frac{x - 20}{5} = -4$ and $x = 10$

(c) $2(x + 5) = 6(x - 1)$ and $x = 4$  
(d) $x^2 - 1 = 2x + 2$ and $x = -1$

(e) $\frac{3(x + 2)}{4} - 1 = 5$ and $x = 2$  
(f) $\frac{3}{4}x - 1 = -\frac{1}{2}x + 9$ and $x = 8$

So, this is no excuse land. If you solve an equation, you should always be able to check to see if your solution is correct. Sometimes, mistakes happen, and it is good to be able to spot them.

**Exercise #4:** Kirk was checking to see if $x = 7$ was a solution to the equation $4x - 3 = 2x + 11$. He concluded that it was not a solution based on the following work. Was he correct?

\[
\begin{align*}
4x - 3 & = 2x + 11 \\
4 \cdot 7 - 3 & = 2 \cdot 7 + 11 \\
4 \cdot 4 & = 2 \cdot 18 \\
16 & = 36 \quad \text{[\text{No}]}.
\end{align*}
\]
EQUATIONS AND THEIR SOLUTIONS
COMMON CORE ALGEBRA I HOMEWORK

FLUENCY

1. Decide if each of the following are equations or expressions. You do not need to solve the equations or evaluate the expressions.

(a) $5x + 13$  
(b) $4x + 3 = 12$  
(c) $\frac{6(x-1)}{4} + 1 = 5$

(d) $3(x+2)^2 - (45)^3$  
(e) $3^2 - 5|2x - 15|$  
(f) $3[(x+2)^2 + 2(x-4)] = 3\sqrt{4(2x+1)}$

2. Determine whether each of the following values for the given variable is a solution to the given equation. Show the calculations that lead to your final conclusions.

(a) $x - 4 = 12$ and $x = 8$  
(b) $\frac{3+x}{4} = 3$ and $x = 9$

(c) $(x+2) - 3(x-4) = 6$ and $x = 4$  
(d) $\frac{1}{3}(x+2) = \frac{-2}{5}(x-9)$ and $x = 4$
APPLICATIONS

3. A disease has three treatments, depending on the percent of the body affected by the disease. Doctors have the treatment down to three stages as follows;

   Stage 1: less than 15%  
   Stage 2: 15-25%  
   Stage 3: 25-50%

For anything more then 50% there is no cure. If the disease is spreading according to the formula \( P = 6d + 5 \) where \( P \) is the percent of the body affected and \( d \) is the number of days, fill out the following chart and explain to a patient what you observed.

<table>
<thead>
<tr>
<th>Days</th>
<th>% of body Affected</th>
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</thead>
<tbody>
<tr>
<td>1</td>
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Explanation of What You Observe:

REASONING

4. Bobby wants to go on a school trip that will cost him $250. He comes up with an equation that represents how much he needs to save each week as follows:

   \[ 25w + 30 = 250, \text{ where } w \text{ is the number of weeks spent saving}. \]

(a) If he has 9 weeks to save will he have enough money to go on the trip? Explain.

(b) He also wants to have $100 spending cash on the trip. He decides to save an extra $10 a week. To do this he changes his original equation as follows;

   \[ 25w + 30 + 10w = 250 + 100, \text{ where } w \text{ is the number of weeks spent saving}. \]

Will nine weeks be enough time now? Show your calculations and Explain.
You spent a lot of time in 8th grade Common Core Math solving linear equations (ones where the variable is raised to the first power only). In fact, the expectation is that you mastered solving linear equations. These types of equations are so essential in mathematics, though, that it pays to work with them more. In today’s lesson, we will be solving linear equations where the variable only occurs once. We will solve these equations by seeing the structure of the expression involving $x$ and using this structure to “undo” what has been done to it.

**Exercise #1:** Consider the equation $5x + 3 = 23$.

(a) List the operations that have been done to the variable $x$ on the left hand side of the equation in the order in which they occurred. 

(b) Solve the equation by reversing what has been done to $x$. Verify that your value of $x$ is a solution by seeing if it makes the equation true.

This is the most basic of all equation solving techniques. It is the most important solving technique in all of mathematics. Be clear on this:

**Solving Equations by Inverse Operations**

If the variable you are solving for shows up only once, identify the operations that have been done on it and reverse them in the opposite order in which they occur.

**Exercise #2:** Find the value of $x$ that solves each equation. In each case, first identify the operations that have occurred to $x$ and reverse them. Show each step.

(a) $\frac{x - 3}{2} + 7 = 23$

What happened to $x$?

(b) $4(x + 1) - 2 = -6$

What happened to $x$?

Now reverse.

Now reverse.
Often equations can be solved in multiple ways. Let’s take a look at the next problem to see an example.

**Exercise #3:** Solve the following equation two different ways. In (a) reverse the operations that have been done to $x$. In (b), apply the distributive property first.

(a) $-2(x - 4) + 8 = 2$ [Reverse the operations]  
(b) $-2(x - 4) + 8 = 2$ [Use the Distributive Prop First]

We should also be prepared to use this technique to solve problems where we must translate between English and mathematics.

**Exercise #4:** Set up equations that translate the following verbal phrases into mathematics and then solve the equations.

(a) Ten less than five times a number results in thirty five. What is the number? Carefully set up an equation, solve it, and check your answer for reasonableness. Watch out! Subtraction is involved.

(b) When three times the sum of a number and seven is increased by ten, the result is four. What is the number? Carefully set up an equation and solve it. Check for reasonableness.
SEEING STRUCTURE TO SOLVE EQUATIONS
COMMON CORE ALGEBRA I HOMEWORK

FLUENCY

1. In the expression \( \frac{x}{5} - 3 \) which is the correct order in which operations have been done to \( x \)?
   (1) \( x \) was divided by 5 and the result was subtracted from 3
   (2) \( x \) had 3 subtracted from it and the result was then divided by 5.
   (3) \( x \) was divided by 5 and 3 was subtracted from the result
   (4) 5 was divided by \( x \) and then 3 was subtracted from the result.

2. Which of the following is the solution to \( 6x + 1 = 4 \)? Show the steps or explain how you found the solution.
   (1) \( x = \frac{7}{6} \)  (3) \( x = \frac{4}{3} \)
   (2) \( x = \frac{1}{2} \)  (4) \( x = \frac{5}{6} \)

3. The solution to \( 5(x - 2) - 6 = 24 \) is which of the following? Show the steps in your solution process.
   (1) \( x = 7 \)  (3) \( x = -3 \)
   (2) \( x = -12 \)  (4) \( x = 8 \)

APPLICATIONS

4. If a number is increased by five and the result is then divided by three, the result is seven. Write an equation that models this verbal description and solve the equation for the number described.

5. Max and his friend Zeke are comparing their ages. They figure out that if they double Max’s age from 3 years ago and add it to Zeke’s current age, the sum is 26. If Zeke is currently 8 years old, determine how old Max currently is.
6. A rectangular area is being fenced in along a river that serves as one side of the rectangle.

(a) Write an equation that relates the amount of fencing, \( F \), needed as a function of the width \( w \) and the length \( l \).

(b) If \( w = 12 \) feet and \( l = 20 \) feet, what is the value of \( F \)?

(c) If we know that the amount of fencing we have available is 120 feet and we want to devote 30 feet to the length, \( l \), then set up an equation to solve for \( w \) and find the width.

**REASONING**

7. Consider the equation \( \frac{5(2x-1)}{3} - 4 = 11 \). This equation looks complicated, but we can unravel all of the operations that have been done to \( x \) to produce the output of 11.

(a) List the operations that have been done to \( x \) and the order in which they have been done.

(b) Reverse the operations from (a) to solve for \( x \).

8. Think about the equation \( 4(3x + 2) = -16 \).

(a) Solve this equation by reversing what has been done to \( x \).

(b) Solve this equation by first distributing the multiplication by 4.
LINEAR EQUATION SOLVING – A REVIEW
COMMON CORE ALGEBRA I

The expectation of the Common Core is that students have mastered solving all types of linear equations in 8th grade Common Core mathematics. In this lesson, we simply present a variety of linear equations for you to practice solving.

**Exercise #1:** Solve each of the following “two-step” linear equations. Keep in mind, this is what we were doing in the last lesson by reversing the operations that had occurred to the variable. Some of these answers will be non-integer rational numbers. Simplify where possible.

(a) \( \frac{x}{3} - 7 = -2 \)  
(b) \( 4x + 3 = -17 \)  
(c) \( 5x + 12 = 87 \)

(d) \( \frac{x + 7}{3} = 2 \)  
(e) \( -6(x - 1) = 18 \)  
(f) \( 8x + 2 = -2 \)

(g) \( \frac{3}{4}x - 5 = 4 \)  
(h) \( -\frac{5}{2}x + 6 = 1 \)  
(i) \( 6x + 3 = -1 \)
For most of what we do the rest of the way, you will be using the distributive property as well as others to solve the problems. Don’t forget our primary technique of solving by reversing the operations that have been done to our variable. This technique is particularly useful when the variable shows up only once!

**Exercise #2**: Solve the following equation for \( x \) by identifying the operations that have been done to \( x \) and reversing them.

\[
\frac{5(x - 3)}{8} + 2 = 7
\]

Reverse them!

Operations?

O.k. Now we move onto problems where this technique is used, but only towards the end. We also need to review how to solve problems where the variable shows up more than once. Since this is review, we will jump right into the most complex scenario.

**Exercise #3**: Consider the equation \( 5(x - 3) + 2x = 4(x + 3) \).

(a) By using the distributive property, write equivalent expressions for both sides of the equation. Show the work below.

(b) Solve the equation for \( x \). Check to make sure the original equation has a true value for the \( x \) you find.

**Exercise #4**: Get more practice on these more complicated equations. Check that your final answer makes the equation true. Generally, use the distributive property when needed.

(a) \( 7(x - 2) - 3(x + 3) = 5(x - 3) + x \)

(b) \( 9 - 6(x + 1) = 2(x - 4) + 27 \)
LINEAR EQUATION SOLVING – A REVIEW
COMMON CORE ALGEBRA I HOMEWORK

FLUENCY

1. Solve the following equations for \( x \) using inverse operations.

(a) \( 7x - 15 = 1 \)
(b) \( \frac{x + 2}{4} = -2 \)
(c) \( \frac{3}{5}x + 2 = 7 \)

2. Solve the equation for \( x \). Check to make sure the original equation has a true value for the \( x \) you find.

(a) \( \frac{5(x + 1) + 4}{6} = 4 \)
(b) \( \frac{5(x - 3)}{8} + 2 = 7 \)

(c) \( -\frac{3}{2}x + 2 = -4 \)
(d) \( 5(x + 1) - 2x = 2(3 + x) \)

(e) \( 3(x - 4) - 2(3x + 4) = 4(3 - x) + 5x + 4 \)
(f) \( \frac{1}{2}(2 - 6x) - 4\left(x + \frac{3}{2}\right) = -(x - 3) + 4 \)
APPLICATIONS

In the real world many scenarios may be modeled with linear equations like the ones you’ve seen so far. Sometimes, though, linear models may not give viable results, and we must interpret the answer we find. To see an example of this, let’s look at the following.

3. A tile warehouse has Inventory at hand and can put in for a back order from a supplier of bundles of tiles. Currently they have 38 tiles of a certain kind in stock, and can only order more in groups of 12 tiles per bundle. The equation that represents this order is as follows;

\[
\text{The number of tiles} = 12b + 38, \text{where } b \text{ is the number of bundles ordered.}
\]

(a) If a customer needs 150 tiles, how many bundles will need to be ordered? Explain how you got your answer. Why do we need to round our answer up in this problem?

(b) If the store likes to keep 30 tiles in stock at all times how many bundles do they need to order now, after selling the 150 tiles to the customer? Think about how many you had left over from the customer who ordered 150 tiles.

REASONING

4. Look through the following work, find the mistake, and circle it. Then, to the side, show the appropriate work.

\[
\frac{-2(x - 3)}{5} = 4
\]

\[
5 \cdot \frac{-2(x - 3)}{5} = 4 \cdot 5
\]

\[
-2(x - 3) = 20
\]

\[
-2x - 6 = 20
\]

\[
-2x - 6 + 6 = 20 + 6
\]

\[
-2x = 26
\]

\[
-2x = 26
\]

\[
-2 = -2
\]

\[
x = -13
\]
Now that we have reviewed how to solve linear equations involving variables on both sides, it is time to take it to another level. The Common Core asks us not only to know the how but also the why. Generally, we justify the steps we take in solving linear equations using the commutative, associative, and distributive properties of real numbers along with the following two properties of equality.

**Properties of Equality**

1. **Additive Property of Equality:** If \( a = b \) then \( a + c = b + c \) (you can add or subtract the same quantity from both sides and retain the equality).

2. **Multiplicative Property of Equality:** If \( a = b \) then \( c \cdot a = c \cdot b \) (you can multiply or divide by the same quantity on both sides and retain the equality).

**Exercise #1:** Consider the equation \( 2x + 9 = 21 \). The steps in solving the equation are shown below. Justify each step.

Step 1: \( 2x + 9 - 9 = 21 - 9 \)  
Justification: _________________________________

Step 2: \( \frac{1}{2} \cdot 2x = \frac{1}{2} \cdot 12 \)  
\( x = 6 \)  
Justification: _________________________________

O.k. That was a reasonably simple two-step equation. Now, let’s go for the full experience.

**Exercise #2:** Consider the equation \( 3(x + 2) - 2(x + 7) = 4x + 7 \). As in the last problem, each step of the solution is shown. Justify each with either a property of equality or a property of real numbers.

Step 1: \( 3x + 6 - 2x - 14 = 4x + 7 \)  
Justification: _________________________________

Step 2: \( 3x + -2x + 6 + -14 = 4x + 7 \)  
Justification: _________________________________

Step 3: \( x(3 - 2) + -8 = 4x + 7 \)  
\( x - 8 = 4x + 7 \)  
Justification: _________________________________

Step 4: \( x - 8 - 4x + 8 = 4x + 7 - 4x + 8 \)  
Justification: _________________________________

Step 5: \( x - 4x - 8 + 8 = 4x - 4x + 7 + 8 \)  
Justification: _________________________________

Step 6: \( x(1-4) = 15 \)  
\( -3x = 15 \)  
Justification: _________________________________

Step 7: \( \frac{-3x}{-3} = \frac{15}{-3} \)  
\( x = -5 \)  
Justification: _________________________________
Strange things can sometimes happen when you solve an equation. Even if every step is justified, results can turn out confusing.

**Exercise #3:** Consider the equation $5x - 3(x + 1) = 2(x + 4)$.

(a) Fill in this missing justifications in the solution of this equation below.

<table>
<thead>
<tr>
<th>Step</th>
<th>Equation</th>
<th>Justification</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>$5x - 3x - 3 = 2x + 8$</td>
<td>The Distributive Property</td>
</tr>
<tr>
<td>2.</td>
<td>$5x - 3x - 3 - 8 = 2x + 8 - 8$</td>
<td></td>
</tr>
<tr>
<td>3.</td>
<td>$x(5 - 3) - 11 = 2x$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$2x - 11 = 2x$</td>
<td></td>
</tr>
<tr>
<td>4.</td>
<td>$2x - 11 - 2x = 2x - 2x$</td>
<td>Additive Property of Equality</td>
</tr>
<tr>
<td>5.</td>
<td>$2x - 2x - 11 = 0$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$-11 = 0$</td>
<td></td>
</tr>
</tbody>
</table>

(b) The final line of this set of manipulations is a very strange statement: $-11 = 0$. Is this a true statement? Could any value of $x$ make it a true statement?

(c) What do you think this tells you about the solutions to this equation (i.e. the values of $x$ that make it true)?

**Exercise #4:** Consider the equation $7x + 2(x + 5) = 9x + 10$.

(a) Show that $x = -5$ and $x = 2$ are both solutions to this equation.

(b) Solve this equation by manipulating each side of the equation as we did above. What does its final “strange” result tell you?

(c) Test your conclusion in (b) by picking a random integer (or really any number) and showing that it is a solution to the equation.
JUSTIFYING STEPS IN SOLVING AN EQUATION
COMMON CORE ALGEBRA I HOMEWORK

FLUENCY

1. Which property justifies the second line in the following solution?

(1) Multiplicative Property of Equality  (3) Distributive  
   \[ 3x + 2 = 8 \]

(2) Associative  (4) Additive Property of Equality  
   \[ 3x + 2 - 2 = 8 - 2 \]

2. What is the solution to the following equation? Show all work.

\[ 3(x + 2) - 2x = -2(x - 3) + 3x \]

(1) No Solutions  (3) \( x = 2 \)

(2) Infinite Solutions  (4) \( x = -3 \)

3. Give a property of real numbers (associative, commutative, or distributive) or a property of equality (addition or multiplication) that justifies each step in the following equation:

\[ 3x + 1 + 2x - 7 = x + 22 \]

(1) \[ 3x + 2x + 1 - 7 = x + 22 \]  
   (1) _________________________________

(2) \[ x(3 + 2) - 6 = x + 22 \]
   \[ 5x - 6 = x + 22 \]  
   (2) _________________________________

(3) \[ 5x - 6 + 6 = x + 22 + 6 \]
   \[ 5x = x + 28 \]  
   (3) _________________________________

(4) \[ 5x - x = x + 28 - x \]  
   (4) _________________________________

(5) \[ x(5 - 1) = 28 \]
   \[ 4x = 28 \]  
   (5) _________________________________

(6) \[ \frac{1}{4} \cdot 4x = \frac{1}{4} \cdot 28 \]
   \[ x = 7 \]  
   (6) _________________________________
APPLICATIONS

4. Antonio just signed up for new phone plan and is comparing his fees to that of his friend Marcus. They both create equations so that they could compare their fees with each other.

   Antonio’s plan: Monthly cost = 3(.75m + 10) + 2.50m – 15 where m is the number of minutes used

   Marcus’s Plan: Monthly cost = 2(1.75m + 12.50) – .75m + 4 where m is the number of minutes used

(a) By setting their monthly cost equal, decide after how many minutes the two plans will cost the same.

(b) Antonio compares his plan to another friend, Brielle’s. Given that both Antonio and Brielle will only be charged for full minutes, is there an amount of time when their two plans cost the same? Explain.

   Brielle’s plan: Monthly cost = 2(1.50m + 12) + m – 4 where m is the number of minutes used

REASONING

5. Without solving the following equations decide whether there will be one solution, no solutions or infinitely many solutions and explain why you think so.

\[
\begin{align*}
3x - 2 &= 3x - 2 \\
2x - 4 &= 2x - 7 \\
3x - 5 &= 6x - 5
\end{align*}
\]
LINEAR WORD PROBLEMS
COMMON CORE ALGEBRA I

Although word problems can often be some of the most challenging for students, they give us great opportunities to refine our understanding of the relationships between quantities and how to manipulate expressions to solve equations. When you solve any real world problem in mathematics you are modeling a physical situation with mathematical tools, such as equations, diagrams, tables, as well as many others.

As we work through these problems, try to make sure to always do the following:

<table>
<thead>
<tr>
<th>MODELING AND SOLVING LINEAR WORD PROBLEMS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Clearly define the quantities involved with common sense variables and let statements.</td>
</tr>
<tr>
<td>2. Use your let statements to write out expressions for quantities that you are interested in.</td>
</tr>
<tr>
<td>3. Carefully translate the information you are told into an equation.</td>
</tr>
<tr>
<td>4. Solve the equation – remember to mentally note the justification for each step.</td>
</tr>
<tr>
<td>5. Check the reasonableness of your answer! This could be the most important, and neglected, step in the modeling/problem solving method.</td>
</tr>
</tbody>
</table>

Let’s start off with a reasonably easy example.

**Exercise #1:** The sum of a number and five more than the number is 17. What is the number?

(a) First experiment with some numbers. This will help you when going to the abstract with variables.

(b) Now, let’s carefully set up let statements and an equation that relates the quantities of interest. Solve the equation for the number.

**Exercise #2:** The difference between twice a number and a number that is 5 more than it is 3. Which of the following equations could be used to find the value of the number, \( n \)? Explain how you arrived at your choice.

1. \( 2n - n + 5 = 3 \)
2. \( n - (2n + 5) = 3 \)
3. \( n + 5 - 2n = 3 \)
4. \( 2n - (n + 5) = 3 \)
The modeling process can become much more complicated when the information becomes more convoluted. Let’s work with one particular age problem next.

**Exercise #3:** Evie and her father are comparing their ages. At the current time, Evie’s father is 36 years older than her. Three years from now, Evie’s father will be five times her age at that point. How old is Evie now?

(a) Before we start to work with setting up variables, expressions, and equations, let’s first do some guess-and-check work. Try a few ages for Evie now, and see if any are correct. Think carefully about the information given in the question.

(b) Set up careful let statements to define expressions that keep track of Evie’s age and her father’s age now and three years from now. Then, set up an equation that summarizes the information in the problem about their ages in five years. Then, solve the equation and check for reasonableness.

**Exercise #4:** Kirk has 12 dollars less than Jim. If Jim spends half of his money, and Kirk spends none, then Kirk will have two dollars more than Jim. How much money did they both start with?
## LINEAR WORD PROBLEMS
### COMMON CORE ALGEBRA I HOMEWORK

### FLUENCY

1. The sum of three times a number and 2 less than 4 times that same number is 15. Which of the following equations could be used to find the value of the number, \( n \)? Explain how you arrived at your choice.

   \[
   (1) \ 3n + 4n - 2 = 15 \quad (3) \ 4n + 3(n - 2) = 15 \\
   (2) \ 3n + 4(n - 2) = 15 \quad (4) \ 3n - 4(n - 2) = 15
   \]

2. Create a let statement for the following examples. Be sure to carefully read the question and figure out exactly what you are looking for. Then, set up an equation that summarizes the information in the problem and solve the equation and check for reasonableness.

<table>
<thead>
<tr>
<th>(a) The sum of 3 less than 5 times a number and the number increased by 9 is 24. What is the number?</th>
</tr>
</thead>
<tbody>
<tr>
<td>(b) Tom is 4 more than twice Andrews age. Sara is 8 less than 5 times Andrews age. If Tom and Sara are twins, how old is Andrew?</td>
</tr>
<tr>
<td>(c) A wireless phone plan costs Eric $35 for a month of service during which he sent 450 text messages. If he was charged an fixed fee of $12.50, how much did he pay per text?</td>
</tr>
<tr>
<td>(d) Daniel is currently 26 years older than his son. In six years he will be three times older than his son. How old are both of them now?</td>
</tr>
</tbody>
</table>
APPLICATIONS

3. There is a competition at the local movie theater for free movie tickets. You must guess all four employees’ ages given a few clues. The first clue is that when added together, their ages total 106 years. Kirk is twice ten years less than the manager’s age, Brian is 12 years younger than twice the manager’s age, and Matt is 6 years older than half the manager’s age. What are all four of their ages? It may help to set up four let statements, one for each employee (including the manager).

REASONING

In some cases the answers you will get won’t make physical sense or need a bit of interpreting. Look at the next example and be careful when you interpret your final solution.

4. Tanisha and Rebecca are signing up for new cellphone plans that only charge for the number of minutes and everything else is included in a monthly fee. Their plans are as follows:

Tanisha’s plan: $0.15 per minute used talking and a $25 monthly fee.
Rebecca’s Plan: $0.10 per minute used talking and a $18.50 monthly fee.

(a) Figure out after how many minutes the two plans will charge the same amount?

(b) Interpret your answer. It may help to read their two plans again and think about which one you would rather pay.
MORE WORK WITH LINEAR EQUATIONS – CONSECUTIVE INTEGER GAMES
COMMON CORE ALGEBRA I

One of the ways we can practice our ability to work with algebraic expressions and equations is to play around with problems that involve **consecutive integers**. Make sure you known what the integers are:

<table>
<thead>
<tr>
<th>The Integers and Consecutive Integers</th>
</tr>
</thead>
<tbody>
<tr>
<td>The <strong>integers</strong> are the subset of the <strong>real numbers</strong>: {...−4, −3, −2, −1, 0, 1, 2, 3,...} (so positive and negative whole numbers).</td>
</tr>
<tr>
<td><strong>Consecutive integers</strong> are any list of integers (however long) that are separated by only 1 unit. Such as:</td>
</tr>
<tr>
<td>1, 2, 3 or 5, 6, 7, 8 or −4, −3, −2 or −10, −9, −8, −7, −6</td>
</tr>
<tr>
<td><strong>Consecutive Evens</strong></td>
</tr>
<tr>
<td>4, 6, 8 or −8, −6, −4, −2 or 14, 16</td>
</tr>
</tbody>
</table>

**Exercise #1**: Let’s work with just two consecutive integers first. Say we have two consecutive integers whose sum is eleven less than three times the smaller integer.

(a) It is important to play around with this problem numerically. So, try a variety of combinations and see if you can find the correct pair of consecutive integers. Be sure to show your calculations.

(b) Now, carefully set up let statements that give expressions for our two consecutive integers. Using these expressions, set up an equation that allows you to find them and solve the equation.
Let’s try some more problems. We always encourage you to play around with numbers before you go to the algebraic set up. The algebra should flow from what you do with numbers, not the other way around.

**Exercise #2:** I’m thinking of three consecutive odd integers. When I add the larger two the result is nine less than three times the smallest of them. What are the three consecutive odd integers?

**Exercise #3:** Three consecutive even integers have the property that when the difference between the first and twice the second is found, the result is eight more than the third. Find the three consecutive even integers.

**Exercise #4:** The sum of four consecutive integers is −18. What are the four integers?
MORE LINEAR EQUATIONS AND CONSECUTIVE INTEGER GAMES
COMMON CORE ALGEBRA I HOMEWORK

FLUENCY

1. Set up let statements for appropriate expressions and using these expressions set up an equation that allows you to find each number described. Be sure to find EACH integer you are looking for.

(a) Find two consecutive integers such that ten more than twice the smaller is seven less than three times the larger.

(b) Find two consecutive even integers such that their sum is equal to the difference of three times the larger and two times the smaller.

(c) Find three consecutive integers such that three times the largest increased by two is equal to five times the smallest increased by three times the middle integer.

(d) Find three consecutive odd integers such that the sum of the smaller two is three times the largest increased by seven.
APPLICATIONS

3. In an opera theater, sections of seating consisting of three rows are being laid out. It is planned so each row will be two more seats than the one before it and 90 people must be seated in each section. How many people will be in the third row?

4. In the same opera theater personal balcony sections with three rows of seating are being mapped as well. In these sections there must be an odd number of seats in each row and each row must have two more seats than the one before it. The last stipulation is that the front row must have one quarter the total seats in the back 2 rows combined. How many seats will be in each row?

REASONING

5. Instead of finding even or odd consecutive integers we could also look for integers that differ by a number other than 2. Find three numbers that each differ by 3 such that 5 times the largest integer is equal to three times the smallest increased by 5 times the middle.

6. What do you think every other even integer means? Set up a let statement that would show this.

7. Find three every other even integers such that the sum of all three is equal to three times the largest decreased by the other two numbers.
At this point we should feel very competent solving linear equations. In many situations, we might even solve equations when there are **no actual numbers given**. Let’s take a look at what we mean in Exercise #1.

**Exercise #1:** Solve each of the following problems for the value of $x$. In (b), write your answer in terms of the unspecified constants $a$, $b$, and $c$.

(a) $5x + 3 = 33$

(b) $ax + b = c$

The rules for solving linear equations (and all equations) don’t depend on whether the constants in the problem are specified or not. The biggest difference in #1 between (a) and (b) is that in (b) you have to leave the results of the intermediate calculation undone.

**Exercise #2:** Solve the following two equations. In letter (b), leave your answer in terms of the constants $a$, $b$, $c$ and $d$.

(a) $\frac{x + 5}{2} - 7 = 3$

(b) $\frac{x + a}{b} - c = d$

Of course, we can have numbers we known (specified constants) thrown into the mix. The most important thing is to know when we can combine and produce a result and when we can’t.

**Exercise #3:** When $2(x - h) + k = 8$ is solved for $x$ in terms of $h$ and $k$, its solution is which of the following? Show the algebraic manipulations you used to get your answer.

(1) $4 + h - k$

(2) $h + 4 - \frac{k}{2}$

(3) $\frac{h}{2} + 8$

(4) $4 - h + k$
Many times this technique is used when we want to rearrange a formula to solve for a quantity of interest.

**Exercise #4:** For a rectangle, the perimeter, \( P \), can be found if the two dimensions of length, \( L \), and width, \( W \), are known.

(a) If a rectangle has a length of 12 inches and a width of 5 inches, what is the value of its perimeter? Include units.

\[
\text{Exercise #5: } \quad \text{Consider the equation } \ ax + b = cx + d \ . \text{ We’d like to solve this equation for } x \ . \text{ Let’s start with the situation where we know the values of } a, b, c \text{ and } d.
\]

(a) Solve: \( 8x + 1 = 5x + 22 \)

(b) Now solve: \( ax + b = cx + d \)

**Exercise #6:** Which of the following solves the equation \( ax - k = 3(x + h) \) for \( x \) in terms of \( a, k, \) and \( h \). Show the manipulations to find your answer.

1. \( \frac{3h + k}{a - 3} \)
2. \( \frac{3a + k}{h - 1} \)
3. \( \frac{k + 3h}{a + 3} \)
4. \( \frac{h + 3}{a + k} \)
SOLVING LINEAR EQUATIONS WITH UNSPECIFIED CONSTANTS
COMMON CORE ALGEBRA I HOMEWORK

FLUENCY

1. When \( \frac{3(x-k)}{w} = 4 \) is solved for \( x \) in terms of \( w \) and \( k \), its solution is which of the following? Show the algebraic manipulations you used to get your answer.

   (1) \( \frac{4}{3}w + k \)  
   (2) \( k - \frac{3w}{4} \)  
   (3) \( k - \frac{4}{3}w \)  
   (4) \( \frac{4}{3} + w - k \)

2. Solve the following equations for \( x \). It may help to make up an equation with numbers and solve it to the side to make sure you are not making any mistakes.

   (a) \( a(x + b) - c = d \)  
   (b) \( \frac{e(x + c)}{b} = 2 \)

   (c) \( rx + qx - d = gc \)  
   (d) \( 2ax - b = cx + d \)

   (e) \( zx = 5g(2x - c) \)  
   (f) \( \frac{ax}{b} + \frac{cx}{d} = e \)
APPLICATIONS

5. In physics the following formula relates your distance above the ground, \( d \), relative to how long, \( t \), an object has been in the air:

\[
d = v_0 t + \frac{1}{2} at^2
\]

(a) Solve the formula for \( a \), the acceleration due to gravity.

(b) Using your manipulated equation, find the value of \( a \) if \( d = 80 \), \( v_0 = 50 \) and \( t = 8 \).

*note: an acceleration towards the ground is negative.

REASONING

4. When traveling abroad many of the units used are different. One of the most common is the unit of temperature namely Fahrenheit versus Celsius. The conversion between the two temperatures is as follows.

\[
C = \frac{5}{9}(F - 32)
\]

(a) Using the formula above convert 50\(^\circ\) Fahrenheit to Celsius.

(b) This conversion formula is very useful if you are given Fahrenheit, but less useful if you know a Celsius temperature. Solve the above equation for Fahrenheit, \( F \), and then convert 50\(^\circ\) Celsius into Fahrenheit. Is there a large difference in Fahrenheit and Celsius?
So far we have concentrated on solving equations. Remember, all solving an equation consisted of was finding values of the variable that made the two expressions equal (in other words made the equation true). We can also judge the truth value of a statement that is in the form of an inequality.

**Exercise #1:** For each inequality, state whether it is true or false.

(a) $7 > 3$  
(b) $0 < 10$  
(c) $9 > 12$  
(d) $4 \leq 4$

(e) $2 \geq 7$  
(f) $3.5 \leq 4.2$  
(g) $256 > 312$  
(h) $1,978 \leq 2,042$

It is quite easy for most students to judge an inequality when the numbers are positive. It becomes more difficult when negative numbers are involved.

**Exercise #2:** Consider the statement $-8 < 4$. Do you think this is true or false? Why? Which is the correct truth value and why?

There are lots of ways to formally define how to determine if one number is greater than another. We will use a graphical definition:

**Inequality Definition**

If we compare any two numbers, say $a$ and $b$, we will say that $a > b$ is true if $a$ lies to the right of $b$ on a standard horizontal number line or above $b$ on a standard vertical number line.

**Exercise #3:** Give the truth values for each of the following statements. Draw a number line to support your work.

(a) $3 > -4$  
(b) $-5 > -3$  
(c) $0 > -6$
So, since we can test the inequality of numbers now, we can also test the inequality of expressions for values of variables. This is identical to checking the truth value of an equation.

**Exercise #4:** Given the inequality \(3(x - 2) \geq 2x + 1\) determine if it is true or false for the following values of \(x\).

(a) \(x = 10\) 
(b) \(x = 5\) 
(c) \(x = 1\) 
(d) \(x = 7\)

Notice that unlike equations, inequalities tend to have many values that make them true. We will eventually discuss that certain inequalities even have an **infinite** number of values for their variables that make them true.

**Exercise #5:** For each of the following inequalities, determine if it is true or false at the given value of the replacement variable.

(a) \(2x + 4 > 4x - 1\) for \(x = 1\) 
(b) \(-3(x + 5) \geq \frac{x + 7}{2}\) for \(x = -3\)

(c) \(x^2 - 10x + 1 < 20 + 5x\) for \(x = -2\) 
(d) \(\frac{2(x - 5) + 1}{3} \leq \frac{x - 2}{9}\) for \(x = 5\)
INEQUALITIES
COMMON CORE ALGEBRA I HOMEWORK

FLUENCY

1. For each inequality, state whether it is true or false.
   
   (a) $3 \leq 8$  
   (b) $8 < 4$  
   (c) $9 > 9$  
   (d) $1,245 \leq 1,245$

   (e) $-12 \geq -6$  
   (f) $3^2 \leq 5^2$  
   (g) $(-3)^2 \geq 3^2$  
   (h) $.99 \leq .98$

2. For each of the following inequalities, determine if it is true or false at the given value of the replacement variable.

   (a) $3x + 2 \leq 2x - 5$ for $x = 8$
   (b) $3x + 2 \leq 2 - 3x$ for $x = -2$

   (c) $(x - 3)^2 > -3(x + 2)$ for $x = 3$
   (d) $\frac{2(3 - 2x)}{5} \leq 2x - 3(x + 1)$ for $x = -1$

   (e) $\frac{x^2 - 4x + 9}{6} > \frac{3x + 1}{5}$ for $x = 3$
   (f) $\left| \frac{-2(5 - x)}{3} \right| \geq \frac{3x - 1}{2}$ for $x = -1$
APPLICATIONS

3. A pressure gage for a boiler allows the boiler to run as long as \( \frac{3(x + 5) - 1}{2} + 4(2x - 3) \leq 125 \) psi, where \( x \) is the pressure reading at the sensor. If the pressure gets too high the machine will shut down to prevent any injuries but it will also cost the company money. Test the following values to see what pressures will be safe for the machine to run at.

<table>
<thead>
<tr>
<th>Pressure readings</th>
<th>Calculations</th>
<th>Safe?</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x = 12 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( x = 13 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( x = 14 )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(b) If the machine cannot run unless the has a pressure above 35 pounds per cubic inch, test to see if a reading of 5 would keep the machine functional.

REASONING

4. Write the appropriate inequality sign (< or >) in the box that will make each of the following true at the given point.

(a) \( 4x + 2 \) \( \underline{1 - 3x} \) for \( x = -2 \)
(b) \( \frac{2x + 1}{-3} \) \( \underline{4(2 - 3x)} \) for \( x = -2 \)

(c) \( 2x^2 + 5 \) \( \underline{|1 - 9x|} \) for \( x = 4 \)
(d) \( \frac{3(2x - 5)}{3} + 2 \) \( \underline{8(3x - 6)} \) for \( x = 5 \)
SOLVING LINEAR INEQUALITIES
COMMON CORE ALGEBRA I

Just as we can solve linear equations by using properties of expressions (commutative, associative, and distributive) and equations (addition and multiplication properties), we can do the same for inequalities. But, we have to make sure we know what those properties are. Let’s test them.

**Exercise #1:** Consider the true inequality $4 < 8$.

(a) If we add 3 to both sides of the inequality, what is the resulting inequality? Is it true?

(b) If we subtract 4 from both sides of the inequality, what is the resulting inequality? Is it true?

(c) If we multiply both sides of the inequality by 2, what is the resulting inequality? Is it true?

(d) If we divide both sides of the inequality by 2, what is the resulting inequality? Is it true?

Hmm… Based on Exercise #1, you might conclude that the truth values of inequalities have the same properties as the truth values for equalities (equations). But there is one huge difference between linear inequalities and linear equations.

**Exercise #2:** Returning to our true inequality $4 < 8$.

(a) If we multiply both sides of the inequality by $-2$, what is the resulting inequality? Is it true?

(b) If we divide both sides of the inequality by $-2$, what is the resulting inequality? Is it true?

**Properties of Inequalities**

1. **The Addition (and Subtraction) Property:** If $a > b$ is true then $a + c > b + c$ is true.

2. **The Multiplication (and Division) Property:** If $a > b$ is true then $c \cdot a > c \cdot b$ will be true if $c$ is a positive number and $c \cdot a < c \cdot b$ will be true if $c$ is a negative number.

**Exercise #3:** Write a true inequality and show that it becomes false when multiplying (or dividing, your choice) each side by a negative.
Now that we know the ways that the truth value of an inequality can remain the same or change, we can solve linear inequalities.

**Exercise #4:** Given the linear inequality $4x - 3 \geq 5$ do the following:

(a) Solve the inequality by applying the properties of inequalities that we found earlier.  
(b) Write 5 numbers that make the final solution true and plot them on the number line below (c).

(c) Now, graph all of the solutions on the number line below (this is called the solution set).

![Number line with solutions](image)

**Exercise #5:** Given the linear inequality $8 - 2x > 16$ do the following:

(a) Rewrite the left hand expression as an equivalent expression using addition.  
(b) Solve the inequality by applying the properties on inequality.

(c) Pick a number that is true based on your solution to (b) and show that it makes the original inequality true.

(d) Graph the solution to the inequality on the number line below.

![Number line with solutions](image)

When we solve inequalities, we will also use the **commutative**, **associative**, and **distributive properties** of numbers (not equations) to write **simpler equivalent expressions** on both sides of the inequality.

**Exercise #6:** Consider the inequality $8(x - 2) - 3(2x + 1) \leq 7x + 4 - 3(x + 1)$.

(a) Use the distributive, commutative, and associative properties of numbers to simply the left and right hand expressions of this inequality.  
(b) Solve the inequality using the properties of inequality and graph the final solution set on a number line that you draw by hand.
SOLVING LINEAR INEQUALITIES
COMMON CORE ALGEBRA I HOMEWORK

FLUENCY

1. Solve the inequality using the properties of inequality and graph the final solution set on the number line provided.

(a) \(5x - 6 \leq 24\)

(b) \(2(5 - x) \leq 12\)

(c) \(6 - 4x > 18\)

(d) \(8x - 6(x - 2) > 20 - 2x\)

(e) \(\frac{3(2x + 2)}{6} > \frac{1}{3}x + 2\)
APPLICATIONS

2. Two siblings Edwin and Rhea are both going skiing but choose different payment plans. Edwin’s plan charges $45 for rentals and $5.25 per lift up the mountain. Rhea’s plan was a bundle where her entire day cost $108.

(a) Set up an inequality that models the number of trips, $n$, up the mountain for which Edwin will pay more than Rhea. Solve the inequality.

(b) What is the greatest amount of trips that Edwin can take up the mountain and still pay less than Rhea? Explain how you arrived at your answer.

REASONING

3. Given $a, b, c, d$ are all positive, solve the following inequalities for $x$.

(a) $ax + b \geq cd$

(b) $\frac{a(x + 2)}{b} > c$

4. If $ax + b > d$ and $a < 0$ then

(1) $x > \frac{d - b}{a}$

(2) $x < \frac{d - b}{a}$

(3) $x < \frac{d}{a} - b$

(4) $x > \frac{d}{a} - b$
Linear inequalities tend to have an infinite amount of values for the replacement variable (typically \( x \)) that solve the inequality. Sometimes, we put two (or more) inequalities together and ask what \( x \) values make both true (AND) and which make either one or the other true (OR). You will deal with AND and OR along with truth values for the remainder of Algebra, so let’s discuss them in an exercise.

**Exercise #1:** Consider each of the following compound (meaning more than one) inequality statements. Determine the truth value of both inequalities and then determine the overall truth value (or at least what you think it is).

(a) \( 7 > 3 \) and \( 2 < 10 \)  
(b) \( 5 < 10 \) and \( 11 > 20 \)

(c) \( -4 < 7 \) or \( 8 < 2 \)  
(d) \( 3 > 6 \) or \( 8 < 5 \)

Most students would correctly judge the truth values of the four examples above correctly. But, there is a strange disconnect in math between our use of the word or and the way it is used in the “real world.” The next exercise will clarify this.

**Exercise #2:** Consider the compound inequality: \( 8 > 2 \) or \( 3 < 10 \).

(a) Determine the truth value of each of the inequalities in this compound inequality.  
(b) What does your intuition tell you the truth value of the compound statement is? What is the mathematical truth value? This is because in mathematics we use the inclusive or.

English Intuition Truth Value:  
Mathematical Truth Value:

**Exercise #3:** Which of the following compound inequalities is false? Explain your reasoning by showing the truth values of each of the individual inequalities.

(1) \( 5 > 2 \) or \( 4 < 1 \)  
(2) \( -6 < 5 \) and \( 7 \geq 7 \)  
(3) \( 10 > 0 \) or \( -3 < 9 \)  
(4) \( -2 \leq 4 \) and \( 5 > 7 \)
Now we can start to judge the truth values of inequalities that involve algebraic expressions and replacement values. Don’t ever forget that:

<table>
<thead>
<tr>
<th><strong>Truth Values for AND and OR</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>1. A compound AND statement will be <strong>true</strong> only if all of the individual statements are true.</td>
</tr>
<tr>
<td>2. A compound OR statement will be <strong>true</strong> if at least one of its individual statements is true.</td>
</tr>
</tbody>
</table>

We would also like to be able to produce number line graphs of compound inequalities. For now, we will stick with a few simple ones.

**Exercise #4:** Determine if each of the following values of \( x \) is in the solution set to the compound inequalities given below?

(a) Is \( x = 2 \) part of the solution set of \( x > -3 \) and \( x < 5 \)? Justify your answer.
(b) Is \( x = -4 \) part of the solution set of \( x \leq -4 \) or \( x > 0 \)? Justify your answer.

(c) Determine if \( x = 1 \) part of the solution set of:

\[
3x + 8 > 9 \quad \text{and} \quad -2x + 10 < 7
\]

Justify.

(d) Determine if \( x = 5 \) part of the solution set of:

\[
2x - 1 < 3 \quad \text{or} \quad \frac{x + 7}{2} = 6
\]

Justify.

**Exercise #5:** On the number lines below, shade in all values of \( x \) that solve the compound inequality. In other words, shade in the compound inequalities solution set. If you need a good place to start, try listing some \( x \) values that make the compound inequalities true.

(a) \( x < 7 \) and \( x \geq -2 \)

List some values:

(b) \( x \geq 5 \) or \( x < -1 \)

List some values:
**FLUENCY**

1. Determine if each of the following statements is true or false. Justify your answer.

   (a) Albany is the capital of New York and New York City is the capital of New York. ______________

   (b) Albany is the capital of New York or New York City is the capital of New York. ______________

   (c) Poughkeepsie is the capital of New York or New York City is the capital of New York. ______________

2. Determine the truth value of each of the following compound inequalities by first determining the truth value of each of the individual inequalities.

   (a) $5 \leq 10$ and $3 < -4$  

   (b) $2 < 7$ or $-20 > -18$

   (c) $-6 < -7$ or $-2 \leq -2$

   (d) $-5 > -8$ and $5 < 8$

3. Which of the following compound inequalities is true? Explain your reasoning by showing the truth values of each of the individual inequalities.

   (1) $5 > 2$ and $4 < 1$  

   (2) $5 \leq 5$ and $-6 \geq -5$

   (3) $-2 > 0$ or $-6 \geq 6$

   (4) $-2 \geq -4$ and $3 > 0$
APPLICATIONS

4. When at a carnival there are height restrictions to go on each ride. Determine which rides each member of this family can go on by filling out the table below:

<table>
<thead>
<tr>
<th></th>
<th>The Swings: ( h &gt; 24 ) and ( h &lt; 70 )</th>
<th>The Twister: ( h &gt; 48 ) or ( h \leq 60 )</th>
<th>Wooden Rollercoaster: ( h &gt; 42 ) and ( h &lt; 72 )</th>
<th>Tea cups: ( h \leq 35 ) or ( h \geq 60 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tracey: ( h = 47 ) inches</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mark: ( h = 70 ) inches</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Marissa: ( h = 28 ) inches</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Which ride can every member go on?

REASONING

5. Determine if each of the following values of \( x \) is in the solution set to the compound inequalities given below? Justify each of your choices by showing your calculations.

(a) \( x = 0 \) for:
\[
3x + 2 \leq 12 \text{ or } 3(x + 1) < -4(3x + 1)
\]

(b) \( x = 2 \) for:
\[
\frac{2(x + 1)}{3} \leq 6 \text{ and } -2(3 - 2x) < 2
\]

(c) \( x = -1 \) for:
\[
3x + 7 < -11 \text{ or } 4 - 2x \leq 18
\]

(d) \( x = 5 \) for:
\[
\frac{2x - 4}{2} \geq 3 \text{ and } \frac{x - 3}{4} = 2
\]
MORE WORK WITH COMPOUND INEQUALITIES
COMMON CORE ALGEBRA I

Compound inequalities are used in mathematics for a variety of purposes. It’s good to get more practice in them, especially when it comes to visualizing what values of \( x \) lie in their solution sets.

**Exercise #1:** Graph each of the following compound inequalities on the number lines provided. For (c) and (d) write the inequalities as a single statement.

(a) \( x < 1 \) or \( x \geq 4 \)

(b) \( x > 7 \) or \( x < -2 \)

(c) \( x > -3 \) and \( x < 5 \)

(d) \( x \leq 9 \) and \( x \geq 0 \)

Inequalities involving AND are almost always universally written as a single inequality because these tend to show us how all values of \( x \) are between two numbers.

**Exercise #2:** Graph each of the following. First, rewrite as two inequalities involving the AND connector.

(a) \( -4 \leq x < 6 \)

(b) \( -5 \leq x \leq 9 \)

**Exercise #3:** For each of the following graphs, write a compound inequality that describes all of the numbers shown graphed.

(a) Compound Inequality: ______________________

(b) Compound Inequality: ______________________
We now can put together our skills at solving inequalities with compound inequalities to write very sophisticated solution sets.

**Exercise #4:** Consider the compound inequality given by:

\[ 6x + 1 \geq 4 \quad \text{and} \quad -2x + 8 > -12 \]

(a) Determine whether each of the following values of \( x \) falls in the solution set to this compound inequality. Show the work that leads to each answer.

\[ x = 5 \quad x = -3 \quad x = 10 \]

(b) Solve the compound inequality and graph its solution on the number line shown below.

![Number line diagram]

A very curious thing happens in the next compound inequality.

**Exercise #5:** Consider the compound inequality shown below:

\[ \frac{1}{2}(x + 4) < 5 \quad \text{or} \quad -2(x - 4) \leq 14 \]

(a) Show that each of the following three values of \( x \) solve the compound inequality.

\[ x = -6 \quad x = 0 \quad x = 8 \]

(b) Solve this compound inequality, graph the solution on the number line. What can you say about the solution set of this inequality?
MORE WORK WITH COMPOUND INEQUALITIES
COMMON CORE ALGEBRA I HOMEWORK

FLUENCY

1. Graph each of the following compound inequalities on the number lines provided. If its an AND statement write the inequalities as a single statement.

   (a) \( x > 5 \) or \( x \leq 3 \)

   (b) \( x \geq -7 \) and \( x < 10 \)

   (c) \( x \leq 3 \) or \( x < -6 \)

   (d) \( x < 3 \) and \( x > -5 \)

2. Graph each of the following. First, rewrite as two inequalities involving the AND connector.

   (a) \(-7 \leq x < 5\)

   Two Inequalities: ________________________

   (b) \(-2 \leq x \leq 6\)

   Two Inequalities: ________________________

3. For each of the following graphs, write a compound inequality that describes all of the numbers shown on the graph.

   (a) Compound Inequality: ________________________

   (b) Compound Inequality: ________________________
4. Consider the compound inequality given by:

\[-2 \leq \frac{1}{2}x + 2 \quad \text{and} \quad \frac{1}{2}x + 2 < 3\]

Solve this compound inequality and graph the solution on the number line. Write the solution set as a single algebraic statement.

5. Consider the compound inequality: \[-7 \leq 2x - 5 < 7\]

(a) Using the skills you have learned today, rewrite the following inequality using the AND connector?

(b) Solve the compound inequality you found in part (a) and graph the solution on the number line. Rewrite your answer as a single statement.

(d) Using the skills above, try and simplify the following inequality. Graph the solution on the number line and rewrite your answer as a single statement.

\[-3 \leq 3x + 3 < 2x + 10\]
INTERVAL NOTATION
COMMON CORE ALGEBRA I

We will often want to talk about continuous segments of the real number line. We’ve already done work with this in the last lesson using what is known as inequality or set-builder notation. Today we will see a very simple way of showing these segments.

**Exercise #1:** For each of the following, graph the portion of the number line described by the inequality and then write the equivalent using interval notation.

(a) \(-3 \leq x \leq 5\)

(b) \(-6 < x < 4\)

(c) \(-4 < x \leq 8\)

(d) \(x \geq 4\)

(e) \(x < 5\)

(f) \(-4 < x\)

One of the great advantages of interval notation is that we essentially need to know a starting value, an ending value and then whether they are included or not.

**Exercise #2:** Which of the following represents the equivalent interval to \(-12 \leq x < 4\)?

(1) \((-12, 4)\)  
(2) \((-12, 4]\)  
(3) \([-12, 4)\)  
(4) \([-12, 4]\)
Eventually, we will use **interval notation** to express solutions sets to inequalities as well as to describe sets of interest to us.

**Exercise #3:** Solve the inequality given below for all values of $x$. Graph the solution on the number line given and state the solution set using interval notation.

$$12 - 4x > 0$$

Intervals express information about particular values of a variable. We can look at the same types of problems from the last lesson, where intervals combine in various ways.

**Exercise #4:** Two inequalities have solution sets given in interval notation below.

- Inequality #1: $(-3, 2)$
- Inequality #2: $(0, 4)$

(a) Write an interval that represents all values that are solutions to both inequalities (AND). Draw number lines to help you think about the solution set.

(b) Write an interval that represents all values that are solutions to either of the inequalities (OR). Draw number lines to help you think about the solution set.

**Exercise #5:** At a hydroelectric plant, Pump #1 is on for all times on the interval $[0, 8)$ and Pump #2 is on for all times in the interval $[4, 18)$. Which of the following represents all times, $t$, when both pumps are on?

(1) $4 \leq t < 8$

(2) $0 \leq t < 18$

(3) $4 < t < 8$

(4) $8 \leq t \leq 18$
INTERVAL NOTATION
COMMON CORE ALGEBRA I HOMEWORK

FLUENCY

1. Write sets using interval notation for the sections of the number lines shown graphed below.

(a) (b)

Equivalent Interval Notation: _______________  Equivalent Interval Notation: _______________

2. For each of the following, graph the portion of the number line described by the inequality and then write the equivalent using interval notation.

(a) \( x > 4 \)  
( b) \( -2 \leq x < 7 \)  

Equivalent Interval Notation: _______________  Equivalent Interval Notation: _______________

(c) \( -3x + 2 < 17 \)  
( d) \( 2x + 5 \geq -6 \)  

Equivalent Interval Notation: _______________  Equivalent Interval Notation: _______________

(e) \( x \geq 3 \) or \( x < 2 \)  
( f) \( x \geq 4 \) and \( x < -4 \)  

Equivalent Interval Notation: _______________  Equivalent Interval Notation: _______________
3. Cookies have to be in the oven between 8 and 12 minutes and brownies have to be in the oven between 9 and 14 minutes. Which of the following represents all times, $t$, when both are in the oven at the same time?

(1) $9 \leq t \leq 12$  
(2) $8 \leq t \leq 14$  
(3) $8 \leq t \leq 14$  
(4) $12 \leq t \leq 14$

**APPLICATIONS**

4. A new office-residential building just opened in Lagrangeville and the contractor is monitoring the water use. For the most part, water is used by the office between the hours of 7 AM – 7 PM and the residential section between 12AM – 9 AM or 3PM – 12AM, including the endpoint times.

(a) Create a compound inequality written in interval notation that represents the hours that both sections (residential and office) are using water at the same time. Graph the solution on the number line given. Assume that 12 AM corresponds to zero and a time such as 3 PM corresponds to 15. As a start, it might help to graph each individual section’s water use and see where they overlap.

(b) If the water heater in the building cannot sustain more than 4 hours of use from both parties at the same time, will there be a period of the day that cold water will start to be produced? Explain

**REASONING**

5. Aidan wrote the interval $(-5, 4]$ and claimed it was equivalent to the graph below. Explain what he did wrong and correct his mistake.
COMMON CORE ALGEBRA I

MODELING WITH INEQUALITIES
COMMON CORE ALGEBRA I

Just as we can solve many real-world problems involving linear equations, there are plenty of situations when an inequality is called for instead. In this lesson, we will practice setting up and solving inequalities based on real-world scenarios.

Exercise #1: A school is taking a field trip with 195 students and 10 adults. Each bus can hold at most 40 students. We need to determine the smallest number of busses needed for the trip.

(a) Using a guess-and-check method, determine the minimum number of busses needed. Show evidence of your thinking.

(b) Let \( n \) be the number of busses taken on the trip. Write and solve an inequality that models this problem based on \( n \).

It is important that you are able to deal with the phrases \textbf{at least} and \textbf{at most}. Let’s try to do some translating.

Exercise #2: Translate each of the following phrases into an inequality. Do not solve.

(a) When three times a number \( n \) is increased by 12, the result is at least 32.

(b) The sum of two consecutive even integers, \( n \) and \( n + 2 \), is at most 8.

Exercise #3: Find all numbers for which five less than half the number is at least seven. Set up an inequality, carefully define expressions and solve the inequality.

Exercise #4: Find all numbers such that twice the sum of the number and eight is at most four. Solve this problem by setting up and solving an inequality.
Let’s try to model a real world scenario with an inequality.

**Exercise #5:** A stadium is steadily filling up with people. It holds at most 2,500 people. Of the 2,500 seats, 350 are reserved for special guests. When the doors open, people fill the seats at a rate of 10 seats per minute.

(a) If \(m\) represents the number of minutes that have gone by, fill out the following chart for how many seats have either been taken or are reserved.

<table>
<thead>
<tr>
<th>(m) (minutes)</th>
<th>Seats Filled</th>
<th>Seats Reserved</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td>350</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
<td>350</td>
<td></td>
</tr>
<tr>
<td>50</td>
<td></td>
<td>350</td>
<td></td>
</tr>
<tr>
<td>100</td>
<td></td>
<td>350</td>
<td></td>
</tr>
</tbody>
</table>

(b) Write an **expression** that calculates the number of seats filled and reserved in terms of the minutes, \(m\), that have passed.

(c) Write an inequality that shows times, in minutes, before the stadium is over-filled. Solve the inequality.

(d) At the rate that people are entering, will any more people be able to find a seat after 4 hours? Justify your yes/no answer.

(e) To cover the cost of the stadium, labor, and other overhead costs, stadium organizers must raise at least $39,000 from ticket sales. If they sell tickets at $25 each, will they have covered the cost if 1,250 tickets are sold?

(f) Let \(n\) represent the number of tickets sold. Write and solve an inequality that represents all values of \(n\) that guarantee the organizers will cover their ticket sales.
MODELING WITH INEQUALITIES
COMMON CORE ALGEBRA I HOMEWORK

FLUENCY

1. Translate each of the following phrases into an inequality, then find the solution set by solving the inequality.

(a) When 4 times a number \( n \) is decreased by 3 it’s at most 21.

(b) When 6 less than 3 times a number is increased by 2, it’s at least 5 times the same number decreased by 8.

(c) Find all numbers such that a third of a number increased by half that number is at least 3 less than that same number.

(d) The sum of 2 consecutive integers is at most the difference between nine times the smaller and 5 times the larger.

(e) The sum of two consecutive even integers is at most seven more than half the sum of the next two consecutive even integers.

(f) A fish tank can hold at most 315 gallons of water. If a hose is filling the fish tank at a rate of 15 gallons every 10 minutes, how many hours can the hose be left on before the tank overflows?
APPLICATIONS

2. A 2.2 GB game is being downloaded onto your laptop. When you have downloaded half a gigabyte, you notice that your computer has been downloading at a rate of .01 GB/min.

(a) Write an inequality that represents at least how many minutes \( m \) it will take to download the whole game.

(b) At this point you also realize your computer only has 2 hours of battery life left and you’ve forgotten your charger. Will there be enough time to download the entire game? Don’t forget you’ve already downloaded some of it.

(c) If, after turning off a few applications, the download speed increases to .015 GB/min will you be able to download the entire game now?

REASONING

3. At an amusement park there’s only enough room for 4500 people to be in it at any time. The manager has also worked out that there needs to be 2800 people in the park to make a profit after all the overhead costs and employee pay. If people are entering the park at a rate of 12 people a minute and there are 850 people in the park currently between how many minutes should the door stay open to let guests in?

(a) Translate the scenario above into a compound inequality involving the number of minutes, \( m \), that the door has been open. Take into account both the fact that there must be a minimum of 2800 people and a maximum of 4500 people.

(b) Rewrite the inequality you found in part (a) using the AND connector and then solve the compound inequality.

(c) Write the solution set as a single statement using interval notation.
UNIT #3

FUNCTIONS

Lesson #1 – Introduction to Functions
Lesson #2 – Function Notation
Lesson #3 – Graphs of Functions
Lesson #4 – Graphical Features
Lesson #5 – Exploring Functions Using the Graphing Calculator
Lesson #6 – Average Rate of Change
Lesson #7 – The Domain and Range of a Function
INTRODUCTION TO FUNCTIONS
COMMON CORE ALGEBRA I

The concept of the function ranks near the top of the list in terms of important Algebra concepts. Almost all of higher-level mathematical modeling is based on the concept. Like most important ideas in math, it is relatively simple:

The Definition of a Function

A function is a clearly defined rule that converts an input into at most one output. These rules often come in the form of: (1) equations, (2) graphs, (3) tables, and (4) verbal descriptions.

Exercise #1: Consider the function rule: multiply the input by two and then subtract one to get the output.

(a) Fill in the table below for inputs and outputs. Inputs are often designated by \( x \) and outputs by \( y \).
(b) Write an equation that gives this rule in symbolic form.
(c) Graph the function rule on the graph paper shown below. Use your table in (a) to help.

<table>
<thead>
<tr>
<th>Input ( x )</th>
<th>Calculation</th>
<th>Output ( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Exercise #2: In the function rule from #1, what input would be needed to produce an output of 17? Why is it harder to find an input when you have an output than finding an output when you have an input?

Exercise #3: A function rule takes an input, \( n \), and converts it into an output, \( y \), by increasing one half of the input by 10. Determine the output for this rule when the input is 50 and then write an equation for the rule.
**Exercise #4:** Function rules do not always have to be numerical in nature, they simply have to return a single output for a given input. The table below gives a rule that takes as an input a neighborhood child and gives as an output the month he or she was born in.

<table>
<thead>
<tr>
<th>Child</th>
<th>Birth Month</th>
</tr>
</thead>
<tbody>
<tr>
<td>Max</td>
<td>January</td>
</tr>
<tr>
<td>Evin</td>
<td>April</td>
</tr>
<tr>
<td>Zeke</td>
<td>May</td>
</tr>
<tr>
<td>Rosie</td>
<td>February</td>
</tr>
<tr>
<td>Niko</td>
<td>May</td>
</tr>
</tbody>
</table>

(a) Why can we consider this rule a function?

(b) What is the output when the input is Rosie?

(c) Find all inputs that give an output of May. Why does this *not* violate the definition of a function even though there are two answers?

Functions are useful because they can often be used to **model** things that are happening in the real world. The next exercises illustrates a function given only in graphical form.

**Exercise #5:** Charlene heads out to school by foot on a fine spring day. Her distance from school, in blocks, is given as a function of the time, in minutes, she has been walking. This function is represented by the graph given below.

(a) How far does Charlene start off from school?

(b) What is her distance from school after she has been walking for 4 minutes?

(c) After walking for six minutes, Charlene stops to look for her subway pass. How long does she stop for?

(d) Charlene then walks to a subway station before heading to school on the subway (a local). How many blocks did she walk to the subway?

(e) How long did it take for her to get to school once she got on the train?
INTRODUCTION TO FUNCTIONS
COMMON CORE ALGEBRA I HOMEWORK

FLUENCY

1. Decide whether each of the following relations is a function. Explain your answer.

<table>
<thead>
<tr>
<th>Input</th>
<th>Outputs</th>
<th>Function?</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a) States</td>
<td>Capitals</td>
<td></td>
</tr>
<tr>
<td>(b) States</td>
<td>Cities</td>
<td></td>
</tr>
<tr>
<td>(c) Families</td>
<td>Pets</td>
<td></td>
</tr>
<tr>
<td>(d) Families</td>
<td>Last names</td>
<td></td>
</tr>
</tbody>
</table>

2. In each of the following examples, use an input-output chart to decide if the following relation is a function.

(a) Consider the following relation: multiply the input by five and then subtract seven to get the output.

<table>
<thead>
<tr>
<th>Input</th>
<th>Calculation</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>-3</td>
<td>None</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>None</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>None</td>
<td></td>
</tr>
</tbody>
</table>

Function? Yes/No

(b) Consider the following table:

<table>
<thead>
<tr>
<th>Input</th>
<th>Calculation</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>None</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>None</td>
<td>3</td>
</tr>
<tr>
<td>3</td>
<td>None</td>
<td>2</td>
</tr>
</tbody>
</table>

Function? Yes/No

(c) Consider the following graph

(d) Consider the following graph

<table>
<thead>
<tr>
<th>Input</th>
<th>Calculation</th>
<th>Output(s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>None</td>
<td>1, 6</td>
</tr>
<tr>
<td>1</td>
<td>None</td>
<td>0, 6</td>
</tr>
<tr>
<td>2</td>
<td>None</td>
<td>0, 6</td>
</tr>
</tbody>
</table>

Function? Yes/No
APPLICATIONS

3. Andrew has a new job at the local pizza store as a delivery boy. The following graph shows one of his deliveries he made. Analyze the graph and answer the following questions.

   (a) How long was the entire trip?

   (b) If he arrived at the house after 4 minutes, how far away was the house from the pizza place?

   (c) Why might Andrew have stopped 3 times for 1 minute?

   (d) Was Andrew’s trip longer going to the house or coming back?

REASONING

4. Given the following scenario, graph a function that would map Liza’s distance away from her house according to the time elapsed.

   Liza has a few items she needs to pick up from a grocery store 8 blocks away. Liza travels as a constant rate of 2 blocks per minute when not stopped at a light. On her way to the grocery store she doesn’t hit any red lights and the trip takes her 4 minutes. She spends 8 minutes in the grocery store and then starts to head home. When she’s halfway home she hits a red light that lasts 3 minutes. After the light ends, she then drives the second half of the way home.
FUNCTION NOTATION
COMMON CORE ALGEBRA I

Since functions are rules that convert inputs (typically x-values) into outputs (typically y-values), it makes sense that they must have their own notation to indicate what the what the rule is, what the input is, and what the output is. In the first exercise, your teacher will explain how to interpret this notation.

Exercise #1: For each of the following functions, find the outputs for the given inputs.

(a) \( f(x) = 3x + 7 \)
   \( f(2) = \)

(b) \( g(x) = \frac{x - 6}{2} \)
   \( g(20) = \)

(c) \( h(x) = \sqrt{2x + 1} \)
   \( h(4) = \)

Function notation can be very, very confusing because it really looks like multiplication due to the parentheses. But, there is no multiplication involved. The notation serves two purposes: (1) to tell us what the rule is and (2) to specify an output for a given input.

Exercise #2: Given the function \( f(x) = \frac{x}{3} + 7 \) do the following.

(a) Explain what the function rule does to convert the input into an output.

(b) Evaluate \( f(6) \) and \( f(-9) \).

(c) Find the input for which \( f(x) = 13 \). Show the work that leads to your answer.

(d) If \( g(x) = 2f(x) - 1 \) then what is \( g(6) \)? Show the work that leads to your answer.
Recall that function rules commonly come in one of three forms: (1) equations (as in Exercise #1), (2) graphs, and (3) tables. The next few exercises will illustrate function notation with these three forms.

**Exercise #3:** Boiling water at 212 degrees Fahrenheit is left in a room that is at 65 degrees Fahrenheit and begins to cool. Temperature readings are taken each hour and are given in the table below. In this scenario, the temperature, \( T \), is a function of the number of hours, \( h \).

<table>
<thead>
<tr>
<th>( h ) (hours)</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>( T(h) ) ( (^\circ F) )</td>
<td>212</td>
<td>141</td>
<td>104</td>
<td>85</td>
<td>76</td>
<td>70</td>
<td>68</td>
<td>66</td>
<td>65</td>
</tr>
</tbody>
</table>

(a) Evaluate \( T(2) \) and \( T(6) \).

(b) For what value of \( h \) is \( T(h) = 76 \) ?

(c) Between what two consecutive hours will \( T(h) = 100 \) ? Explain how you arrived at your answer.

**Exercise #3:** The function \( y = f(x) \) is defined by the graph shown below. It is known as **piecewise linear** because it is made up of **straight line segments**. Answer the following questions based on this graph.

(a) Evaluate each of the following:

\[
  f(1) = \quad f(5) = \\
  f(-3) = \quad f(0) = 
\]

(b) Solve each of the following for all values of the input, \( x \), that make them true.

\[
  f(x) = 0 \quad f(x) = 2 
\]

(c) What is the largest output achieved by the function? At what \( x \)-value is it hit?
FUNCTION NOTATION
COMMON CORE ALGEBRA I HOMEWORK

**FLUENCY**

1. Given the function \( f \) defined by the formula \( f(x) = 2x + 1 \) find the following:
   
   (a) \( f(4) \) \hspace{1cm} (b) \( f(-5) \) \hspace{1cm} (c) \( f(0) \) \hspace{1cm} (d) \( f\left(\frac{1}{2}\right) \)

2. Given the function \( g \) defined by the formula \( g(x) = \frac{x-5}{2} \) find the following:
   
   (a) \( g(9) \) \hspace{1cm} (b) \( g(0) \) \hspace{1cm} (c) \( g(3) \) \hspace{1cm} (d) \( g(17) \)

3. Given the function \( f \) defined by the formula \( f(x) = x^2 - 4 \) find the following:
   
   (a) \( f(3) \) \hspace{1cm} (b) \( f(-4) \) \hspace{1cm} (c) \( f(0) \) \hspace{1cm} (d) \( f(-2) \)

4. If the function \( f(x) \) is defined by \( f(x) = \frac{x}{2} - 6 \) then which of the following is the value of \( f(10) \)?

   (1) \(-1\) \hspace{1cm} (3) \(14\)
   
   (2) \(2\) \hspace{1cm} (4) \(7\)

5. If the function \( f(x) = 2x - 3 \) and \( g(x) = \frac{3}{2}x + 1 \) then which of the following is a true statement?

   (1) \( f(0) > g(0) \) \hspace{1cm} (3) \( f(8) = g(8) \)
   
   (2) \( f(2) = g(2) \) \hspace{1cm} (4) \( g(4) < f(4) \)
6. Based on the graph of the function \( y = g(x) \) shown below, answer the following questions.

(a) Evaluate each of the following. Illustrate with a point on the graph.

\[
\begin{align*}
g(-2) &= \\
g(0) &= \\
g(3) &= \\
g(7) &= 
\end{align*}
\]

(b) What values of \( x \) solve the equation \( g(x) = 0 \)? These are called the **zeros of the function**

(c) How many values of \( x \) solve the equation \( g(x) = 2 \)? How can you illustrate your answer on the graph? Remember, we are not looking for the exact \( x \)-values, only **how many solutions**.

**APPLICATIONS**

6. Physics students drop a ball from the top of a 100 foot high building and model its height above the ground as a function of time with the equation \( h(t) = 100 - 16t^2 \). The height, \( h \), is measured in feet and time, \( t \), is measured in seconds. Be careful with all calculations in this problems and remember to do the exponent (squaring) first.

(a) Find the value of \( h(0) \). Include proper units. (b) Calculate \( h(2) \). Does our equation predict that the ball has hit the ground at 2 seconds? Explain.

**REASONING**

7. If you knew that \( f(-4) = 8 \), then what \((x, y)\) coordinate point must lie on the graph of \( y = f(x) \)? Explain your thinking.
Graphs of Functions
Common Core Algebra I

Graphs are one of the most powerful ways of visualizing a function’s rule because you can quickly read outputs given inputs. You can also easily see features such as maximum and minimum output values. Let’s review some of those skills in Exercise #1.

**Exercise #1:** Given the function \( y = f(x) \) defined by the graph below, answer the following questions.

(a) Find the value of each of the following:

\[
f(4) = \quad f(-1) =
\]

(b) For what values of \( x \) does \( f(x) = -2 \)? Illustrate on the graph.

(c) State the minimum and maximum values of the function.

So, if we can read a graph to produce outputs (\( y \)-values) if we are given inputs (\( x \)-values), then we should be able to reverse the process and produce a graph of the function from its algebraically expressed rule.

**Exercise #2:** Consider the function given by the rule \( g(x) = 2x + 3 \).

(a) Fill out the table below for the inputs given.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( 2x + 3 )</th>
<th>( (x, y) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-3</td>
<td>\</td>
<td></td>
</tr>
<tr>
<td>-2</td>
<td>\</td>
<td></td>
</tr>
<tr>
<td>-1</td>
<td>\</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>\</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>\</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>\</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>\</td>
<td></td>
</tr>
</tbody>
</table>

(b) Draw a graph of the function on the axes provided.
Never forget that all we need to do to translate between an equation and a graph is to plot input/output pairs according to whatever rule we are given. Let’s look at a simple non-linear function next.

**Exercise #3:** Consider the simplest quadratic function \( f(x) = x^2 \). Fill out the function table below for the inputs given and graph the function on the axes provided.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( x^2 )</th>
<th>((x, y))</th>
</tr>
</thead>
<tbody>
<tr>
<td>-3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>-2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>-1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Sometimes the function’s rule gets all sorts of funny and can include being piecewise defined. These functions have different rules for different values of \( x \). These separate rules combine to make a larger (and more complicated rule). Let’s try to get a feel for one of these.

**Exercise #4:** Consider the function given by the formula \( f(x) = \begin{cases} 
2x + 6 & x < 0 \\
6 - x & x \geq 0
\end{cases} \). Your teacher will help you understand the notation of this function.

(a) Evaluate each of the following:

\[
\begin{align*}
  f(4) &= \\
f(-3) &= 
\end{align*}
\]

(b) Fill out the table below for the inputs given. Keep in mind which formula you are using.

<table>
<thead>
<tr>
<th>( x )</th>
<th>Rule/Calculation</th>
<th>((x, y))</th>
</tr>
</thead>
<tbody>
<tr>
<td>-3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>-2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>-1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
1. Using the graph of the function \( f(x) \) shown below, answer the following questions.

(a) Find the value of each of the following:

\[
\begin{align*}
  f(-7) &= \quad f(0) &= \\
  f(4) &= \quad f(9) &= 
\end{align*}
\]

(b) For how many values of \( x \) does \( f(x) = 5 \)? Illustrate on the graph.

(c) What is the y-intercept of this relation?

(d) State the maximum and minimum values the graph obtains.

(e) Explain why the graph above represents a function.

2. Consider the function \( f(x) = 3(2-x) - 2 \). Fill out the function table below for the inputs given and graph the function on the axes provided.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( 3(2-x) - 2 )</th>
<th>( (x, y) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>-1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
APPLICATIONS

3. The following graph represents the cost of a phone plan after a certain number of text messages used in a month. Analyze the graph to answer the following questions.

(a) How much would you have to pay if you used:
   - 500 text messages ____________
   - 1800 text messages ____________

(b) Interpret \( f(1400) = 60 \)

(c) What might have caused the graph to begin increasing at 800 text messages?

REASONING

4. Consider the following relationship given by the formula \( f(x) = \begin{cases} 3 - 2x & x \leq 1 \\ 2x - 1 & x > 1 \end{cases} \).

(a) Evaluate each of the following:
   - \( f(5) = \)
   - \( f(-2) = \)

(b) Carefully evaluate \( f(1) \).

(c) Fill out the table below for the inputs given. Keep in mind which formula you are using.

<table>
<thead>
<tr>
<th>( x )</th>
<th>Rule/Calculation</th>
<th>((x, y))</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(e) What is the minimum value of the function? Circle the point that indicates this value on the graph.
GRAPHICAL FEATURES AND TERMINOLOGY
COMMON CORE ALGEBRA I

There is a lot of terminology associated with the graph of a function. Many of the terms have names that are descriptive, but still, work is needed to master the ideas.

Exercise #1: The function \( y = f(x) \) is shown graphed below over the interval \(-7 \leq x \leq 7\).

(a) Find the maximum and minimum values of the function.
State the values of \( x \) where they occur as well.

(b) What is the \( y \)-intercept of the function? Explain why a function cannot have more than one \( y \)-intercept.

(c) Give the \( x \)-intercepts of the function. These are also known as the function’s zeroes because they are where \( f(x) = 0 \).

(d) Would you characterize the function as increasing or decreasing on the domain interval \(-5 \leq x \leq -1\)? Explain your choice.

(e) one additional interval over which the function is increasing and one over which it is decreasing.

Increasing: __________________
Decreasing: __________________

(f) The following points are known as turning points. Each can be classified as a relative maximum or a relative minimum. State which you think each is.

\((-5, -1)\) \hspace{1cm} \((-1, 7)\) \hspace{1cm} \((2, 2)\) \hspace{1cm} \((5, 5)\)
relative minimum \hspace{1cm} relative minimum \hspace{1cm} relative minimum \hspace{1cm} relative minimum
or \hspace{1cm} or \hspace{1cm} or \hspace{1cm} or
relative maximum \hspace{1cm} relative maximum \hspace{1cm} relative maximum \hspace{1cm} relative maximum
Let’s get some more practice with **piecewise defined functions** and mix in our **function terminology** while we are at it.

**Exercise #2:** Consider the **piecewise linear** function given the equation \( f(x) = \begin{cases} x + 3 & x \leq 1 \\ 6 - 2x & x \geq 1 \end{cases} \).

(a) Create a table of values for this function below over the interval \(-4 \leq x \leq 4\). Then create a graph on the axes for this function.

<table>
<thead>
<tr>
<th>( x )</th>
<th>Rule/Calculation</th>
<th>((x, y))</th>
</tr>
</thead>
<tbody>
<tr>
<td>-4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>-3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>-2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>-1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(b) State the **zeroes of the function**.

(c) State the function’s **y-intercept**.

(d) Give the interval over which the function is increasing. Give the interval over which it is decreasing.

Increasing: _________________

Decreasing: _________________

(e) Give the coordinates of the one turning point and classify it as either a relative maximum or relative minimum.

(f) Use your graph to find all solutions to the equation \( f(x) = 2 \). Illustrate your solution graphically and find evidence in the table you created.

(g) State the interval over which this function is positive. How can you tell this quickly from the graph?
GRAPHICAL FEATURES
COMMON CORE ALGEBRA I HOMEWORK

FLUENCY

1. The function \( y = f(x) \) is shown graphed below over the interval \(-8 \leq x \leq 8\).

   (a) Evaluate each of the following:
   \[
   f(-2) = \quad f(8) = \\
   f(-8) = \quad f(4) =
   \]

   (b) Find all the relative maximum and minimum values of the function. State the values of \( x \) where they occur as well.

   (c) What are the absolute maximum and absolute minimum values of the function? At what \( x \)-values do they occur?

   (d) What are the \( x \) and \( y \)-intercept(s) of the function? List each of the following as an ordered pair \((x, y)\).
   \[
   x\text{-intercept(s):} \quad \text{y-intercept(s):}
   \]
   (zeroes)

   (e) Give an interval over which the function is increasing. Give an interval over which it is decreasing.
   
   Increasing: 
   Decreasing:

   (f) Use your graph to find all solutions to the equation \( f(x) = 3 \). Illustrate your solution graphically.

   (g) Is the function positive or negative on the interval \(-1 < x < 3\)? How can you quickly tell?
APPLICATIONS

2. The following graph shows the height, \( h \), above the ground of a toy rocket \( t \) seconds after it was fired. Use the graph of \( h(t) \) to answer the following questions.

(a) What was the maximum height the rocket reached? After how many seconds?

(b) How many seconds was the rocket in flight?

(c) Interpret \( h(2) = 90 \).

(d) Give the interval for \( t \) over which the height of the rocket is decreasing.

REASONING

3. On the following set of axis, create the graph of a function \( f(x) \) with the following characteristics:

Passes through the points, 

\((-8,0), (5,-2) \) and \((8,3)\)

Has an absolute maximum at \( f(-4) = 5 \)

Has an absolute minimum at \( f(2) = -6 \)

Decreasing on the interval on the interval \(-4 \leq x \leq 2\)
EXPLORING FUNCTIONS USING THE GRAPHING CALCULATOR
COMMON CORE ALGEBRA I

Graphing calculators are powerful tools in our exploration of functions and the rules that define them. Because calculators are so good at doing calculations, it is fairly easy to have them evaluate expressions that are the rules for generating the outputs for the functions. Throughout this entire lesson, we will assume that you have a calculator that can do the following:

**GRAPHING CALCULATOR ESSENTIALS**

1. A TABLE APP
2. A GRAPHING APP

We can use our calculator to help us produce tables that are very useful in plotting graphs and exploring functions.

**Exercise #1:** Consider the linear function \( f(x) = \frac{1}{2}x + 2 \). Do the following by using your graphing calculator’s table function.

(a) Evaluate \( f(-6) \), \( f(0) \) and \( f(8) \).

(b) Explore the table to determine the value of \( x \) for which \( f(x) = 11 \).

(c) Use the table to fill out the following table and graph the function on the grid for the interval \(-6 \leq x \leq 6\).

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
<th>((x, y))</th>
</tr>
</thead>
<tbody>
<tr>
<td>-6</td>
<td>( _ )</td>
<td>( _ )</td>
</tr>
<tr>
<td>-4</td>
<td>( _ )</td>
<td>( _ )</td>
</tr>
<tr>
<td>-2</td>
<td>( _ )</td>
<td>( _ )</td>
</tr>
<tr>
<td>0</td>
<td>( _ )</td>
<td>( _ )</td>
</tr>
<tr>
<td>2</td>
<td>( _ )</td>
<td>( _ )</td>
</tr>
<tr>
<td>4</td>
<td>( _ )</td>
<td>( _ )</td>
</tr>
<tr>
<td>6</td>
<td>( _ )</td>
<td>( _ )</td>
</tr>
</tbody>
</table>

(d) Graph the linear function \( g(x) = 5 - x \) on the same set of axes and find where the two lines intersect.

(e) Show that the point that you found in (d) is a solution to both equations:

\[
y = \frac{1}{2}x + 2 \quad \text{and} \quad y = 5 - x
\]
The calculator can do the heavy lifting with the calculations, while we examine the results. Always be careful when entering algebraic expressions on your calculator. Let’s take a look at a **quadratic function** using the graphing calculator.

**Exercise #2:** Consider the function \( y = (x - 1)^2 - 4 \) over the interval \(-1 \leq x \leq 4\). Do the following with the use of tables on your graphing calculator.

(a) Create a table of values for this function over the specified interval.

(b) Create a sketch of this function over this interval. Verify by examining the graph that your calculator produces.

(c) What are the function’s minimum and maximum values on this interval?

(d) Over what interval is the function negative?

(e) For your graph, state the interval over which the function is increasing.

(f) How can this graph help to solve the equation \((x - 1)^2 - 4 = -3\)? Can you solve this by looking at your table?

**Exercise #3:** Which of the following is a point where \( y = \frac{3}{2}x + 7 \) and \( y = -5x - 6 \) intersect?

(1) \((0, 7)\)  
(2) \((-1, -1)\)  
(3) \((-2, 4)\)  
(4) \((2, 10)\)
Explain functions using the graphing calculator

Fluency

1. Consider the function \( g(x) = 3x^2 + 2x - 4 \). Evaluate the following using your graphing calculator.
   
   (a) \( g(-2) = \)
   
   (b) \( g(0) = \)
   
   (c) \( g(4) = \)
   
   (d) \( g(15) = \)

2. Given the function \( f(x) = x^2 - 2x + 1 \), fill in the missing values in the table then using the table graph the function on the grid for the interval. Use your calculator.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
<th>( (x, y) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td></td>
<td>((-1,4))</td>
</tr>
<tr>
<td>0</td>
<td></td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>9</td>
<td></td>
</tr>
</tbody>
</table>

3. Which of the following values of \( x \) will make the equation \( 3(x - 2)^2 - 4 = 23 \) true? Show the table on your calculator that justifies your choice.
   
   (1) \( x = 1 \)  
   (2) \( x = 4 \)  
   (3) \( x = 5 \)  
   (4) \( x = 0 \)
APPLICATIONS

4. Profits for the upcoming year for a shipping company have been quantified and put into the equation
\[ P(x) = \frac{1}{2}(x-2)^2 - 8 \] where \( x \) is the number of packages shipped in thousands and \( P(x) \) is the corresponding profit in millions of dollars.

(a) Use your calculator to fill out the following table and graph the function on the grid for the interval \( 0 \leq x \leq 10 \).

<table>
<thead>
<tr>
<th>( x )</th>
<th>( P(x) )</th>
<th>( (x, y) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(b) Over what interval is \( P(x) < 0 \)? What does this interval represent?

(c) Evaluate \( P(0) \). What might this stand for? (d) Explore the table to determine the value of \( x \) for which \( P(x) = 0 \). What might this stand for?

REASONING

5. After placing an equation into his calculator Rob got the following table. He then determines that \( x = 6 \) when \( f(x) = -4 \). Is he correct? Explain.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-4</td>
<td>6</td>
</tr>
<tr>
<td>-2</td>
<td>3</td>
</tr>
<tr>
<td>0</td>
<td>-1</td>
</tr>
<tr>
<td>2</td>
<td>-4</td>
</tr>
</tbody>
</table>
AVERAGE RATE OF CHANGE
COMMON CORE ALGEBRA I

Functions are rules that give us outputs when we supply them with inputs. Very often, we want to know how fast the outputs are changing compared to a change in the input values. This is referred to as the average rate of change of a function.

**Exercise #1:** Max and his younger sister Evie are having a race in the backyard. Max gives his sister a head start and they run for 20 seconds. The distance they are along in the race, in feet, is given below with Max’s distance given by the function \( m(t) \) and Evie’s distance given by the function \( e(t) \).

(a) How do you interpret the fact that \( m(12) = 30 \)? Illustrate your response by using the graph.

(b) If both runners start at \( t = 0 \), how much of a head start does Max give his little sister? How can you tell?

(c) Does Max catch up to his sister? How can you tell?

(d) How far does Max run during the 20 second race? How far does Evie run? What calculation can you do to find Evie’s distance?

(e) How fast do both Evie and Max travel? In other words, how many feet do each of them run per second? Express your answers as decimals and attach units.

**Max’s Speed**  
(FEET PER SECOND)  
**Evie’s Speed**  
(FEET PER SECOND)
In the first exercise we were calculating the rate that the function’s output (y-values) were changing compared to the function’s input (or x-values). This is known as finding the average rate of change of the function. You might think you’ve seen this before. And you have.

**Exercise #2:** Finding the average rate of change is the same as finding the ____________ of a line.

There is, of course, a formula for finding average rate of change. Let’s get it out of the way.

### Average Rate of Change

For the function $y = f(x)$, the average rate that $f(x)$ changes from $x = a$ to $x = b$ is given by:

$$
\frac{f(b) - f(a)}{b - a} = \frac{\text{how much the y-values have changed}}{\text{how much the x-values have changed}}
$$

**Exercise #3:** Consider the function given by $f(x) = x^2 + 3$. Find its average rate of change from $x = -1$ to $x = 3$. Carefully show the work that leads to your final answer.

**Exercise #4:** The function $h(x)$ is given in the table below. Which of the following gives its average rate of change over the interval $2 \leq x \leq 6$? Show the calculations that lead to your answer.

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) $-\frac{3}{2}$</td>
<td>(3) $-\frac{7}{6}$</td>
</tr>
<tr>
<td>(2) $\frac{6}{4}$</td>
<td>(4) $-1$</td>
</tr>
<tr>
<td>$x$</td>
<td>$h(x)$</td>
</tr>
<tr>
<td>0</td>
<td>10</td>
</tr>
<tr>
<td>2</td>
<td>9</td>
</tr>
<tr>
<td>4</td>
<td>6</td>
</tr>
<tr>
<td>6</td>
<td>3</td>
</tr>
</tbody>
</table>

**Exercise #5:** Frances is selling glasses of lemonade. The function $g(t) = \frac{t^2 + 4}{2}$ models the number of glasses she has sold, $g$, after $t$-hours. What is the average rate at which she is selling lemonade between $t = 2$ and $t = 6$ hours. Include proper units in your answer.
AVERAGE RATE OF CHANGE
COMMON CORE ALGEBRA I HOMEWORK

FLUENCY

1. Consider the function given by \( f(x) = 9 - x^2 \). Find its average rate of change between the following points. Carefully show the work that leads to your final answer.
   (a) \( x = 0 \) to \( x = 3 \)  
   (b) \( x = -1 \) to \( x = 5 \)  
   (c) \( x = -2 \) to \( x = 2 \)

2. The function \( f(x) \) is given in the table below. Find its average rate of change between the following points. Show the calculations that lead to your answer.
   (a) \( x = -3 \) to \( x = 1 \)  
   (b) \( x = 0 \) to \( x = 4 \).

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-3</td>
<td>7</td>
</tr>
<tr>
<td>0</td>
<td>-2</td>
</tr>
<tr>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>4</td>
<td>-8</td>
</tr>
</tbody>
</table>

3. The function \( f(x) \) is given in the graph below. Find its average rate of change between the following points. Show the calculations that lead to your answer.
   (a) \( x = -6 \) to \( x = 4 \)  
   (b) \( x = -2 \) to \( x = 2 \).
APPLICATIONS

4. The following table shows the number of points the Arlington girls team scored in their last basketball game where \( t \) is the time passed in minutes and \( f(t) \) the total number of points scored after \( t \) minutes.

<table>
<thead>
<tr>
<th>( t )</th>
<th>( f(t) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>8</td>
<td>30</td>
</tr>
<tr>
<td>16</td>
<td>48</td>
</tr>
<tr>
<td>24</td>
<td>55</td>
</tr>
<tr>
<td>32</td>
<td>64</td>
</tr>
</tbody>
</table>

(a) What was the average rate they were shooting in the first half of the game? Be sure to include proper units in your answer.

(b) What was their average rate over the whole game?

(c) Given your answers above which half of the game do you feel they had a better rate of scoring? Explain.

REASONING

5. Consider the function given by \( f(x) = 6x + 5 \).

(a) Find its average rate of change from \( x = 1 \) to \( x = 5 \).

(b) Find its average rate of change from \( x = -2 \) to \( x = 6 \).

(c) The average rate of change for this function is always 6 (as you should have found in the first two parts of the problem). What type of function has a constant average rate of change? What do we call this average rate of change in this case? Search the Internet if needed.
THE DOMAIN AND RANGE OF A FUNCTION
COMMON CORE ALGEBRA I

Ultimately, all functions do is convert inputs into outputs. So, each function has two sets associated with it. Those things that serve as inputs and those things that serve as outputs. These sets are given names.

**THE DOMAIN AND RANGE OF A FUNCTION**

1. The **domain of a function** is the set of all inputs for which the function rule can give an output.
2. The **range of a function** is the set of all outputs for which there is an input that results in them.

**Exercise #1:** Consider the function \( y = f(x) \) shown on the graph below.

(a) Evaluate each of the following:
\[
 f(-3) = \quad f(1) = \quad f(3) =
\]

(b) Can the function rule, given by the graph, give you a value when \( x = 5 \)? If so, what is it? If not, why not?

(c) Is \( x = 5 \) in the **domain** of the function?

(d) Give two other values of \( x \) that are not in the **domain** of the function.

(e) Circle the following \( y \)-values that are in the **range** of the function? Show evidence on your graph.
\[
 y = 0 \quad y = 6 \quad y = -1
\]
\[
 y = 3 \quad y = -5 \quad y = 4
\]

(f) Write the domain and range of this function using a single inequality.

**DOMAIN**

**RANGE**
Exercise #2: Given the function \( f(x) = \frac{x}{2} - 3 \) and the domain shown below, fill in the range. Write the set in roster notation.

Range: ________________

Exercise #3: Which of the following values is not in the domain of the function \( f(x) \) shown below? Illustrate your thinking by marking points on the graph.

(1) \(-3\)  
(2) \(-4\)  
(3) \(5\)  
(4) \(0\)

Exercise #4: Consider the piecewise linear function given by the formula \( f(x) = \begin{cases} \frac{-(x+2)}{2} & -4 \leq x \leq 2 \\ 4x-10 & 2 \leq x \leq 4 \end{cases} \). Determine the function’s range.
**THE DOMAIN AND RANGE OF A FUNCTION**

**COMMON CORE ALGEBRA I HOMEWORK**

**FLUENCY**

1. In each of the following, state the domain and range; then decide if it’s a function or not. Be sure to explain using words such as input, output, domain and range!

   (a) ![Graph of a function](image1.png)

   **Domain:** ________________________  
   **Range:** ________________________  
   **Function (yes/no):** ________________________

   (b) ![Graph of a function](image2.png)

   **Domain:** ________________________  
   **Range:** ________________________  
   **Function (yes/no):** ________________________

2. Consider the piecewise linear function given by the formula 
   
   \[ f(x) = \begin{cases} 
   2 - 3x & -1 \leq x \leq 1 \\
   x - 2 & 1 < x \leq 3 
   \end{cases} \]

   Determine the function’s domain and range. Draw a graph of the function to fully justify your answer. Use tables on your calculator to help graph.

   ![Graph of a piecewise linear function](image3.png)
APPLICATIONS

3. The following graph represents the height above the ground versus time at a resort as Thomas rides his favorite ski slope.

(a) State the domain and, in your own words, what the domain represents.

(b) State range and, in your own words, what the range represents.

(c) What might Thomas have been doing for the interval $0 \leq t \leq 2$? What was his average rate of change? Use proper units in your answer.

(d) What might Thomas have been doing for the interval $2 \leq t \leq 6$? What was his average rate of change? Use proper units in your answer and compare to what you found in (c).

REASONING

4. The graph below represents the height of a ball over the interval $0 \leq t \leq 8$. After how many seconds was the ball 12 feet off of the ground? Explain your answer.

What does your answer indicate about the range of this function?
UNIT #4

LINEAR FUNCTIONS AND ARITHMETIC SEQUENCES

Lesson #1 – Proportional Relationships
Lesson #2 – Unit Conversions
Lesson #3 – Nonproportional Linear Relationships
Lesson #4 – More Work Graphing Linear Functions (Lines)
Lesson #5 – Writing Equations in Slope-Intercept Form
Lesson #6 – Modeling with Linear Functions
Lesson #7 – More Linear Modeling
Lesson #8 – Strange Lines – Vertical and Horizontal
Lesson #9 – Absolute Value and Step Functions
Lesson #10 – The Truth About Graphs
Lesson #11 – Graphs of Linear Inequalities
Lesson #12 – Introduction to Sequences
Lesson #13 – Arithmetic Sequences
You’ve studied proportional relationships in previous courses, but they are the basis of all linear functions, so we will take a lesson to recall their particulars.

**PROPORTIONAL RELATIONSHIPS**

Two variables have a **proportional relationship** if their respective values are always in the same ratio (they have the same relative size to one another). In equation form, if the two variables are $x$ and $y$ then:

$$\frac{y}{x} = \text{constant}$$

**Exercise #1:** At a local farm stand, six apples can be bought for four dollars. Determine how much it would cost to buy the following amounts of apples. Round to the nearest cent, when necessary.

(a) a dozen apples  
(b) 20 apples  

(c) If $c$ is the total cost of apples and $n$ is the number of apples bought, write a proportional relationship between $c$ and $n$. Solve this equation for the variable $c$.

(d) Graph the relationship below.

(e) According to the graph, $c(15) = 10$. Illustrate this on your graph. How do you interpret $c(15) = 10$ in terms of apples and money spent?
**Exercise #2:** If Jenny can run 5 meters in 2 seconds, then which of the following gives the distance, \(d\), she can run over a span of \(t\)-seconds going at the same constant rate? Show the work that leads to your answer.

1. \(d = \frac{2}{5}t\)
2. \(d = 5t + 2\)
3. \(d = 2t + 5\)
4. \(d = \frac{5}{2}t\)

Exercise #2 illustrates one of the most important proportional relationships, that of distance traveled compared to time traveled at a constant rate. Let’s work some more with this.

**Exercise #3:** Erika is driving at a constant rate. She travels 120 miles in the span of 2 hours.

(a) If Erika travels at the same rate, how far will she travel in 3 hours?

(b) Write a proportional relationship between the distance \(D\) that Erika will drive over the time \(t\) that she travels, assuming she continues at this same rate. Solve the proportion for \(D\) as a function of \(t\).

(c) What is the value of the proportionality constant? What are its units?

(d) How much time will it take for Erika to travel 150 miles.

(e) Graph \(D\) as a function of \(t\) on the axes at the right.

(f) What does the constant of proportionality, from (c) represent about this graph? Explain your thinking.
1. A nutrition company is marketing a low-calorie snack brownie. A serving size of the snack is 3 brownies and has a total of 50 calories.

(a) Determine how many calories 6 brownies would have.  
(b) Determine how many calories 21 brownies would have.

(c) Determine how many calories 14 brownies would have. Round to the nearest calorie.  
(d) If $c$ represents the number of calories and $b$ represents the number of brownies, write a proportional relationship involving $c$ and $b$ and solve it for $c$.

(e) Graph the proportional relationship you found in part (d) on the grid shown.

(f) Using the graph, what is the smallest whole number of brownies a person would need to eat in order to consume 125 calories? Illustrate on your graph.

(g) Algebraically determine the number of brownies a person would need to eat in order to consume 300 calories.
2. A local animal feed company makes its feed by the ton, which is 2000 pounds. They want to include a medication in the feed. Each cow needs 300 milligrams (mg) of this medication a day and each cow consumes 15 pounds of the feed per day. If there are 1,000 milligrams in a gram, how many grams of the medication should the feed company add for each ton of feed they produce?

3. Kwan is driving at a constant speed. After \(1\frac{1}{4}\) hours he has driven a total distance of 90 miles.
   (a) How far will Kwan drive in 2 hours at this rate?
   (b) If \(D\) represents the distance Kwan has driven in miles and \(t\) represents the time he has been driving, in hours, then write an equation for \(D\) in terms of \(t\).
   (c) Use your equation from (b) to determine how far Kwan drives in 15 minutes.
   (d) Kwan is driving a total of 234 miles. How long will his trip take him, to the nearest tenth of an hour, assuming he travels at this constant rate? Use proper units.

**REASONING**

**Unit rates** are proportions where we compare the change in one variable to a change of one unit in the other variable. When we typically report speeds in miles per hour, that is a unit rate. A speed of 65 miles per hour should be interpreted as 65 miles traveled per 1 hour of time. When we say that fat has 9 calories per gram, that is a unit rate because we are comparing 9 calories to 1 gram.

4. Convert each of these into unit rates. Some will be decimal unit rates.
   (a) 24 feet per 3 seconds
   (b) 30 pounds per 8 boxes
   (c) 50 calories per 20 chips
UNIT CONVERSIONS
COMMON CORE ALGEBRA I

Units are amazingly important in mathematics, science, and engineering. They are how we decide on what constitutes the number 1 (i.e. 1 gallon, 1 pound, etcetera). We often need to convert from one unit to another in practical problems. In this situation we can almost always use proportional reasoning to do the job.

Exercise #1: John has traveled a total of 4.5 miles. If there are 5,280 feet in each mile, how many feet did John travel? Set up and solve a proportion for this problem. Also, do the problem by multiplying by a ratio.

Exercise #2: If there are exactly 2.54 centimeters in each inch, how many centimeters are in one foot? Show the work that leads to your answer.

Sometimes it is helpful to be able to convert so that a rate makes more sense. Take a look at the next problem.

Exercise #3: A bathtub contains 14.5 cubic feet of water. If water drains out of the bathtub at a rate of 4 gallons per minute, then how long will it take, to the nearest minute, to drain the bathtub? There are 7.5 gallons of water per cubic foot. Show the work that leads to your answer.

Exercise #4: The mile and the kilometer are relatively close in size. Can you convert 1 mile into an equivalent in kilometers? Here’s what I’ll give you. There are 2.54 centimeters in an inch, 5,280 feet in a mile, 100 centimeters in a meter, and 1000 meters in a kilometer. All else you should be able to do for yourself. Round your answer to the nearest tenth of a kilometer. This takes quite a string of multiplications, but you can do it!
We can also convert the ratio of two quantities, or rates, into different units if need be.

Exercise #5: One interesting conversion is from a speed expressed in feet per second to a speed in miles per hour. We sometimes think better in miles per hour because that is how the speeds of our cars are measured.

(a) Convert a speed of 45 miles per hour into feet per second given that there are 5,280 feet in a mile.

(b) The current fastest human is Usain Bolt, from Jamaica. In 2009, Usain ran 100 meters in a blazing 3.22 feet per second average speed. How does this compare to a typical car driving speed?

Exercise #6: A local factory has to add a liquid ingredient to make their product at a rate of 13 quarts every 5 minutes. How many gallons per hour of the ingredient do they need to add? Show the work that leads to your answer.

Exercise #7: A tractor can plant a field at a rate of 2.5 acres per 5 minutes. If a mammoth farm measuring 4 square miles needs planting, how long will it take in hours to plant the field? There are 640 acres in a square mile. Determine your answer to the nearest hour. If the tractor can run 8 hours a day, what is minimum number of days it will take to plant the farm?
UNIT CONVERSIONS
COMMON CORE ALGEBRA I HOMEWORK

APPLICATIONS

1. How many centimeters are there in 1 yard if there are 2.54 centimeters per inch? Show your work and express your answer without rounding.

2. How close are a meter and a yard? Convert 1 meter into yards by using the fact that there are 100 centimeters in a meter, 2.54 centimeters in an inch, 12 inches in a foot, and 3 feet in a yard. Round your answer to the nearest tenth of a yard.

3. If there are 1000 grams in a kilogram and 454 grams in a pound, how many pounds are there per kilogram? Round to the nearest tenth of a pound.

4. Water is flowing out of an artesian spring at a rate of 8 cubic feet per minute. How many minutes will it take for the water to fill up a 300 gallon tank. There are 7.5 gallons of water per cubic foot. Show or explain how you arrive at your answer.
5. A high school track athlete sprints 100 yards in 15 seconds.
   (a) Determine the number of feet per second the runner is traveling at. Show your work.
   (b) If there are 5280 feet in a mile and 3600 seconds in an hour, determine the runner’s speed in miles per hour. Round to the nearest tenth.

6. A cafeteria is trying to scale a small pancake recipe up in order to feed a group of tourists. The recipe feeds 6 people and the cafeteria is trying to feed 75. The recipe calls for 4 cups of flour and \( \frac{1}{2} \) cups of milk and \( \frac{1}{2} \) cup of sugar (as well as some other minor ingredients such as baking powder).
   (a) One 10 pound bag of flour contains 38 cups of flour. Will it be enough for this recipe? Justify.
   (b) If one 10 pound bag of flour contains 38 cups of flour, how many pounds of flour will be needed for this recipe? Round to the nearest tenth of a pound.
   (c) If there are 4 cups in a quart and 4 quarts in a gallon, will we need more or less than a gallon of milk for this recipe?
   (d) The cafeteria has a 1.5 kilogram bag of sugar. If a cup of sugar weighs 0.5 pounds and there are 2.2 pounds per kilogram, does the cafeteria have enough sugar to make this recipe?
   (e) If the original recipe made 14 pancakes and the cafeteria plans to charge $0.50 per pancake, how much money will they make if they sell all of the pancakes made for the 75 people?
NON-PROPORTIONAL LINEAR RELATIONSHIPS
COMMON CORE ALGEBRA I

In this unit’s first lesson, we saw the simplest type of linear relationship, one where the two variables are proportional to one another. In that case, recall:

**PROPORTIONAL RELATIONSHIPS**

The variables $x$ and $y$ are proportional if: $\frac{y}{x} = k$ or $y = kx$. In other words, one variable is always a constant multiple of the other.

But, there are lots of linear relationships (ones that when graphed would form a line) that are not proportional. How can we relate them with an equation?

**Exercise #1:** Consider the linear function $f(x)$ shown below.

(a) Evaluate $f(-2)$ and $f(1)$. What two coordinate points do these function values correspond to?

(b) Calculate the average rate of change of $f$ from $x = -2$ to $x = 1$. This is also known as what quantity for this line?

(c) Is there a proportional relationship between $x$ and $y$? How can you check?

(d) Based on your 8th grade coursework, what relationship does exist between the two variables? Write this equation and check it for the points from (a).
In general, what is always proportional on a linear function is the **change in \( y \)** to the **change in \( x \)**, also known as the **line’s slope**. This gives rise to what is known as the **slope-intercept** form of a line.

**The Slope-Intercept Form of a Linear Function**

Given a linear function, \( f(x) \), it can be expressed in equation form by:

\[
 f(x) = y = mx + b
\]

where \( m = \) average rate of change = slope = \( \frac{\Delta y}{\Delta x} \) and \( b = y \)-intercept of the line.

**Exercise #2**: Given the linear function \( g(x) = \frac{1}{2}x + 1 \) do the following.

(a) Create a limited table of values to help graph the function.

(b) Create a graph of the function on the axes below.

(c) Illustrate the slope of the function graphically.

(d) Circle the graph’s \( y \)-intercept.

**Exercise #3**: Use information about the slope and \( y \)-intercept to graph \( y = -\frac{3}{5}x + 4 \) on the grid. Pick two points off the graph and calculate the average rate of change and verify that it is equal to the slope.
**COMMON CORE ALGEBRA I, UNIT #4 – LINEAR FUNCTIONS AND ARITHMETIC SEQUENCES – LESSON #3**

**NONPROPORTIONAL LINEAR RELATIONSHIPS**

**COMMON CORE ALGEBRA I HOMEWORK**

**FLUENCY**

1. For the linear function $g(x) = 7x - 2$, which of the following is true?
   
   (1) It has a slope of 7 and a $y$-intercept of $-2$.
   (2) It has a slope of $-2$ and a $y$-intercept of 7.
   (3) It has a slope of $7x$ and a $y$-intercept of $-2$.
   (4) It has a slope of $-2$ and a $y$-intercept of $7x$.

2. Which of the following represents the average rate of change of the function $g(x) = \frac{3}{2}x + 1$ over the interval $-2 \leq x \leq 8$?
   
   (1) $\frac{9}{7}$
   (2) $\frac{5}{4}$
   (3) $\frac{2}{3}$
   (4) $\frac{3}{2}$

3. What is the equation of the line shown in the graph below?

   (1) $y = 2x + 4$
   (2) $y = 2x - 2$
   (3) $y = \frac{1}{2}x - 2$
   (4) $y = \frac{1}{2}x + 4$

4. Which of the following is the equation of a line whose slope is 3 and which passes through the point $(2, 7)$?

   (1) $y = 3x + 7$
   (2) $y = 7x + 3$
   (3) $y = 3x + 1$
   (4) $y = 7x - 7$

5. Which of the following is the equation of a line that passes through the points $(0, 8)$ and $(6, 4)$? Use of grid is optional.

   (1) $y = -\frac{2}{3}x + 8$
   (2) $y = \frac{3}{2}x + 6$
   (3) $y = -\frac{4}{5}x + 4$
   (4) $y = \frac{1}{2}x + 8$
6. Graph each of the following linear functions on the grid provided and label with their equations. For each, create a table without the use of your calculator to maintain fluency with operation facts. Show your table. In the first problem, the x-values are given. In others, you will have to choose them. Always include $x = 0$.

(a) $f(x) = 2x + 3$

<table>
<thead>
<tr>
<th>$x$</th>
<th>-5</th>
<th>-2</th>
<th>0</th>
<th>2</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f(x)$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(b) $g(x) = -\frac{1}{2}x - 1$

<table>
<thead>
<tr>
<th>$x$</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>$g(x)$</td>
<td></td>
</tr>
</tbody>
</table>

(c) $h(x) = 5 - x$

<table>
<thead>
<tr>
<th>$x$</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>$h(x)$</td>
<td></td>
</tr>
</tbody>
</table>

7. State the values of the slope and the y-intercept for each of the following linear functions. Then, use this information to create graphs of the functions on the grid below. Label each with its equation.

(a) $y = \frac{2}{3}x - 4$

Slope: _______  y-intercept: _______

(b) $y = -\frac{5}{2}x + 7$

Slope: _______  y-intercept: _______

(c) $y = 3x - 2$

Slope: _______  y-intercept: _______

(d) $y = -x + 3$

Slope: _______  y-intercept: _______
It is critical that you are able to graph lines and understand graphs of lines. Try the first exercise as a warm up.

**Exercise #1:** Four lines are graphed on the set of axes below. Write the number of the line beside each of the correct equations.

<table>
<thead>
<tr>
<th>EQUATION</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>( y = -\frac{2}{3}x + 3 )</td>
<td></td>
</tr>
<tr>
<td>( y = x + 5 )</td>
<td></td>
</tr>
<tr>
<td>( y = -2x - 7 )</td>
<td></td>
</tr>
<tr>
<td>( y = 2x - 3 )</td>
<td></td>
</tr>
</tbody>
</table>

Recall that if a line is written in the form \( y = mx + b \), then it is relatively easy to graph, especially if \( m \) and \( b \) are reasonably easy to work with. A quick review from the previous lesson.

**Exercise #2:** On the grid below, graph the equation \( y = \frac{3}{2}x - 3 \). First, identify its slope and \( y \)-intercept to help you with the graph.

Slope: __________

\( y \)-intercept: __________

**Exercise #3:** Write down two points this line passes through and use them to calculate the average rate of change of this function.
Sometimes linear equations are not written in a form that makes it easy to determine the slope and the \( y \)-intercept. It is important to be able to rearrange these formulas in order to quickly identify these linear parameters.

**Exercise #4:** Consider the linear equation given by \( 2y - 6x = 12 \).

(a) Steps are shown below that rearrange this equation. Justify each step with a property of equality or a property of numbers.

1. \( 2y - 6x + 6x = 12 + 6x \)
2. \( 2y = 6x + 12 \)
3. \( \frac{2y}{2} = \frac{6x + 12}{2} \)
4. \( y = \frac{6x}{2} + \frac{12}{2} \)

\[ y = 3x + 6 \]

(b) Identify the slope and the \( y \)-intercept of this line.

**Exercise #5:** Rearrange each of the following linear equations into \( y = mx + b \) form and identify the slope and the \( y \)-intercept.

(a) \( 3y - 3x = 15 \)

(b) \( 2y + 5x = -8 \)

(c) \( x - 3y = 6 \)

(d) \( 6x - 4y = -20 \)
MORE WORK GRAPHING LINEAR FUNCTIONS
COMMON CORE ALGEBRA I HOMEWORK

FLUENCY

1. Four lines are shown graphed. Place the number of the line next to the equation that most appropriately models it.

   \[ y = \frac{2}{3}x + 5 \]  
   \[ y = x - 3 \]  
   \[ y = -\frac{3}{4}x + 3 \]  
   \[ y = -\frac{1}{2}x - 4 \]

2. The two lines \( y = ax + b \) and \( y = cx + d \) are shown graphed below. The values of \( a, b, c, \) and \( d \) are not given, but properties of them can be inferred from the graph. Circle the pair of values below that could be equal? Explain.

   \( b \) and \( d \)  
   \( a \) and \( d \)  
   \( a \) and \( c \)

   Explain:

3. Which of the following is true about the linear function \( 2y + x = 18 \).

   (1) It has a slope of 2 and a \( y \)-intercept of 18.
   (2) It has a slope of \( -2 \) and a \( y \)-intercept of 9.
   (3) It has a slope of \( -\frac{1}{2} \) and a \( y \)-intercept of 9.
   (4) It has a slope of \( \frac{1}{2} \) and a \( y \)-intercept of 18.

4. For the line \( 2y - 6x = 10 \), for every unit increase in \( x \) which of the following is true?

   (1) \( y \) decreases by 6  
   (2) \( y \) increases by 3  
   (3) \( y \) increases by 2  
   (4) \( y \) decreases by 10
5. Rewrite each of the following linear equations in equivalent \( y = mx + b \) (slope-intercept) form. Identify the slope and the \( y \)-intercept and then graph on the grid given. Label each line with its original equation.

(a) \( 2y - 3x = 10 \)

Slope: ________  \( y \)-intercept: ________

(b) \( x + 2y = 6 \)

Slope: ________  \( y \)-intercept: ________

(c) \( 3y + 12 = 5x \)

Slope: ________  \( y \)-intercept: ________

6. What are the coordinates of the one point shared in common between the two linear functions given below?

\( y = 2x - 2 \)

\( 3y + x = 15 \)

Do you remember what this type of problem is called from 8\(^{th}\) grade Common Core Mathematics?
One skill that we need to become **fluent** at in Algebra I is creating the equation of a linear function. We will concentrate on learning how to form equations in the **slope-intercept form** that we have been working with.

**The Slope-Intercept Form of a Linear Function**

Given a linear function, \( f(x) \), it can be expressed in equation form by:

\[
f(x) = y = mx + b
\]

where the two **parameters** are \( m = \text{average rate of change} = \text{slope} = \frac{\Delta y}{\Delta x} \) and \( b = \text{y-intercept} \) of the line.

**Exercise #1**: Consider the linear function whose graph is shown below.

(a) Determine an equation in the form \( y = mx + b \) for this line.

(b) Test your equation for the value \( x = 2 \).

When the y-intercept is an **integer**, such as in the last exercise, it is fairly easy to get the **exact relationship** between \( x \) and \( y \). Let’s try another graphical problem where the y-intercept is not an **integer**.

**Exercise #2**: Find the equation of the linear function shown in slope-intercept form. Test your equation for \( x = -4 \).
We need to also be able to find the equation for a linear function if we know two points that lie on it. Notice that this means we have to determine the value of the **two parameters** with two pieces of information.

**Exercise #3:** Find the equation of the line that passes through each of the following pairs of points in \( y = mx + b \) form.

(a) \((2, 5)\) and \((5, 17)\)  
(b) \((-2, 5)\) and \((2, 3)\)  
(c) \((-1, 11)\) and \((4, -4)\)  
(d) \((3, 4)\) and \((12, 19)\)

**Exercise #4:** A car is traveling along a straight road. After one hour, the car is 72 miles from Chicago. After three hours, the car is 188 miles from Chicago. Determine an equation for the distance, \(d\), the car is from Chicago after \(h\)-hours if the relationship between \(d\) and \(h\) is linear.
**WRITING EQUATIONS IN SLOPE-INTERCEPT FORM**

**COMMON CORE ALGEBRA I HOMEWORK**

**FLUENCY**

1. Each of the following lines has a slope and $y$-intercept that can be determined by examining the graph. For each, state the slope, the $y$-intercept, and then write the equation in $y = mx + b$ form (slope-intercept form).

(a) ![Graph](Image)

Slope: __________

$y$-intercept: __________

Equation: _________________

(b) ![Graph](Image)

Slope: __________

$y$-intercept: __________

Equation: _________________

2. Each of the following lines has a slope that can be determined by examining the graph. Use another point on the line to solve for the exact $y$-intercept. Then, state the equation of the line.

(a) ![Graph](Image)

Slope: _________________

Solve for $y$-intercept:

Equation: ________________

(b) ![Graph](Image)

Slope: _________________

Solve for the $y$-intercept:

Equation: ________________
3. Find the equation of the line that passes through each of the following pairs of points in \( y = mx + b \) form.

(a) \((1, 7)\) and \((4, 22)\)  
(b) \((-2, 13)\) and \((2, 3)\)

(c) \((4, 6)\) and \((10, 0)\)  
(d) \((0, -10)\) and \((16, 2)\)

APPLICATIONS

4. A steady snow fall is coming down outside. Prestel decides to measure the depth of the snow on the ground. After 4 hours, the snow is at a depth of 9 inches and after 8 hours it is at a depth of 14 inches.

(a) Express the information given in this problem as two coordinate pairs, \((h, d)\), where \(h\) is the number of hours and \(d\) is the depth of snow.

(b) Find the slope of the line that passes through these two points. What are its units?

(c) Find the equation of the line that passes through the two points in \(d = mh + b\) form.

(d) What was the depth when the snowfall began \((h = 0)\)? What would the depth be after 12 hours?
MODELING WITH LINEAR FUNCTIONS
COMMON CORE ALGEBRA I

When we use equations to model real-world phenomena we often look to linear models first because they are the easiest to use and understand. We can now use our skills from the last few lessons to model real-world linear phenomena.

Don’t ever forget these two facts about linear models:

**CRITICAL LINEAR MODEL FACTS**

All linear models in the form \( y = mx + b \) have two parameters, the slope, \( m \), and the \( y \)-intercept, \( b \):

1. The slope, \( m \), always tells us how fast the output is changing relative to the input.
2. The \( y \)-intercept, \( b \), always tells us “how much” we start with, or the output’s starting value (at \( x = 0 \)).

**Exercise #1:** Jannine has $450 in her savings account at the beginning of the year. She places money in the account at the rate of $5 per week. We want to model the amount of money she has in savings, \( s \), as a function of the number of weeks she has been saving, \( w \).

(a) Fill out the table below for some of the number of weeks. Show the calculations that result in your answer.

<table>
<thead>
<tr>
<th>Number of weeks, ( w )</th>
<th>Calculation</th>
<th>Amount in Savings, ( s )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(b) Use information in the givens or in the table to write an equation for the savings, \( s \), as a linear function of the weeks she has been saving, \( w \).

(c) If Jannine saves for exactly one year, what is the range in her savings over the year? Show how you arrived at your answer.

(d) Why would it not make sense to evaluate \( s(6.5) \)? In other words, what types of numbers belong in the domain of this linear function?

(e) Use two points from the table to verify that the rate of change of the function is 5. How do the units show up in the calculation?
Sometimes the information we have about the linear relationship does not include the starting value. Let’s take a look at that type of situation.

**Exercise #2:** Kirk is driving along a long-road at a constant speed. He is driving directly towards Denver. He knows that after 2-hours of driving he is 272 miles from Denver. After 3 and a half hours he is 176 miles from Denver.

(a) Summarize the information given in the problem as two ordered pairs, where the number of hours, $h$, is the input and the distance from Denver, $D$, is the output.

(b) Calculate $\frac{\Delta D}{\Delta h} = \frac{D(3.5) - D(2)}{3.5 - 2}$. Include proper units in your answer.

(c) You should have found that the rate of change was negative. Why is it? Explain what is physically happening to result in this negative rate of change.

(d) Assuming the relationship is linear (which it would be at a constant speed), write an equation for the distance $D$ as a linear function of the number of hours, $h$.

(e) How far did Kirk start from Denver? Show the work that leads to your answer.

(f) After how many hours will Kirk arrive in Denver? Show the work that leads to your answer.

**Exercise #3:** Amanda is walking away from a light pole at a rate of 4 feet per second. If she starts at a distance of 6 feet from the light pole, which of the following gives her distance, $d$, from the light pole after walking for $t$-seconds?

1. $d = 4t + 6$
2. $d = \frac{3}{2} t$
3. $d = 6t + 4$
4. $d = -6t + 4$
MODELING WITH LINEAR FUNCTIONS
COMMON CORE ALGEBRA I HOMEWORK

FLUENCY

1. Water is building up in a bathtub. After 2 minutes there are 12 gallons of water and after 4 minutes, there are 20 gallons of water. What is the average rate at which water is entering the bathtub from $t = 2$ to $t = 4$ minutes? Show how you calculated the rate.

   (1) 8 gallons per minute (3) 10 gallons per minute
   (2) 6 gallons per minute (4) 4 gallons per minute

2. Francisco is saving money in an account. At the beginning of the year, he has $56 in savings and puts in another $4 per week. Which of the following equations models the amount of savings, $s$, as a function of the number of weeks, $w$, Francisco has been saving?

   (1) $s = 4w + 56$ (3) $s = 56w + 4$
   (2) $s = \frac{w}{4} + 56$ (4) $s = \frac{w}{56} + 4$

APPLICATIONS

3. Maria charges $15 for every 2 hours that she babysits. Answer the following questions based on this information.

   (a) How much would Maria charge for working for 5 hours?

   (b) Fill out the table below for the amount that Maria makes as she babysits and graph the relationship on the grid provided.

<table>
<thead>
<tr>
<th>Hours Worked, $h$</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
<th>10</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>Amount, $a$, in $</td>
<td>$</td>
<td>15</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

   (c) Write an equation for the amount, $a$, that Maria makes as a function of the number of hours, $h$, that she babysits. Keep in mind that Maria will make $0 for babysitting for 0 hours.
4. The temperature is falling outside at a steady rate of 4 degrees Fahrenheit every hour. If the temperature starts at 68 Fahrenheit do the following.

(a) Fill out the table below for the outside temperature during the time it is cooling down.

<table>
<thead>
<tr>
<th>Time Cooling, $t$, (hours)</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Temperature, $F$, (Fahrenheit)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(b) Write a linear equation that relates the Fahrenheit temperature, $F$, to the time in hours, $t$, that it has been falling.

(c) According to your equation, what is the temperature when $t = 2.75$ hours?

(d) If this cooling continues at this constant rate, how many hours will it take for the temperature to reach the freezing point of water? Show your work.

5. The population of deer in a park is growing over the years. The table below gives the population found in a survey by local wildlife officials.

<table>
<thead>
<tr>
<th>Year</th>
<th>2000</th>
<th>2003</th>
<th>2006</th>
<th>2009</th>
</tr>
</thead>
<tbody>
<tr>
<td>Deer Population</td>
<td>168</td>
<td>216</td>
<td>264</td>
<td>312</td>
</tr>
</tbody>
</table>

(a) Find the average rate that the deer population is changing over each time interval below:

From 2000 to 2003

From 2003 to 2006

From 2006 to 2009

(b) Why does this calculation indicate a linear relationship?

(c) If $t$ stands for the number of years since 2000, write an equation for the deer population, $p$, as a function of $t$.

(d) What does your model predict the deer population to be in the year 2014?

(e) How many years will it take for the deer population to reach 500? Round to the nearest year.
**MORE LINEAR MODELING**
**COMMON CORE ALGEBRA I**

Although it can be challenging, it is critically important that students who exit Algebra I have a good ability to deal with linear relationships. In this lesson we get more practice modeling linear phenomena.

**Exercise #1:** A warehouse is keeping track of its inventory of cardboard boxes. At the beginning of the month, they had a supply of 1,275 boxes left. They use boxes at a rate of 75 per day.

(a) How many boxes are left after 10 days? Show the calculation that leads to your answer.

(b) Which of the following linear equations correctly models the number of boxes left, \( n \), after \( d \)-days?

\[
\begin{align*}
(1) \quad & n = 75d + 1275 \\
(2) \quad & n = 1275d + 75 \\
(3) \quad & n = 1275 - 75d \\
(4) \quad & n = 75d - 1275
\end{align*}
\]

(c) If the warehouse needs to order more boxes when their supply reaches 150, how many days can they wait?

(d) If after 5 days, they start using boxes at a rate of 90 per day, how many days will it be before they run out of boxes? Show the work that leads to your answer.

We want to feel very comfortable with quickly and accurately determining linear models. Keep in mind that the two most important aspects of any linear model are its rate of change (slope) and its starting value (y-intercept).

**Exercise #2:** The cost, \( c \), in dollars of running a particular factory that produces \( w \)-widgets can be modeled using the linear function.

\[
c(w) = 1.25w + 2175
\]

(a) How do you interpret the fact that \( c(100) = 2300 \) ?

(b) Give a physical interpretation for the two parameters in this equation, 1.25 and 2175.
**Exercise #3:** Biologists estimate that the number of deer in Rhode Island in 2003 was 1,028, and in 2008 it had grown to 1,488. Biologists would like to model the deer population, \( p \), as a function of the years, \( t \), since 2000.

(a) Represent the information we have been told as two coordinate points. Be careful to know what your values of time are for each year.  

(b) Calculate \( \frac{\Delta p}{\Delta t} \) from 2003 to 2008. Include proper units in your answer.

(c) Give a physical interpretation of the value you found in part (b).

(d) Determine a linear relationship between the deer population, \( p \), and the years since 2000, \( t \).

(e) How many deer does this model predict were in Rhode Island in the year 2000? What does this represent about the linear function?

(f) How many deer does the model predict for Rhode Island now?

**Exercise #4:** Water is draining out of a bathtub such that the volume still left, \( g \)-gallons, is shown as a function of the number of minutes, \( m \), it has been draining.

\[
\begin{array}{c|cccc}
  m, \text{ minutes} & 0 & 2 & 4 & 6 \\
  g(m), \text{ gallons} & 62 & 28 & 12 & 5 \\
\end{array}
\]

(a) Calculate the average rate of change of \( g \) over the interval \( 0 \leq t \leq 2 \). Include proper units.

(b) Calculate the average rate of change of \( g \) over the interval \( 2 \leq t \leq 4 \). Include proper units.

(c) Why can we say that the relationship between \( m \) and \( g \) is not linear?
MORE LINEAR MODELING
COMMON CORE ALGEBRA I HOMEWORK

APPLICATIONS

1. A water tank is being filled by pumps at a constant rate. The volume of water in the tank $V$, in gallons, is given by the equation:

$$V(t) = 65t + 280,$$

where $t$ is the time, in minutes, the pump has been on

(a) At what rate, in gallons per minute, is the water being pumped into the tank?

(b) How many gallons of water were in the tank when the pumps were turned on?

(c) What is the volume in the tank after two hours of the pumps running?

(d) The pumps will turn off when the volume in the tank hits 10,000 gallons. To the nearest minute, after how long does this happen?

2. A solar lease customer built up an excess of 6,500 kilowatt hours (kwh) during the summer using his solar panels. When he turned his electric heat on, the excess began to be used up at a rate of 50 kilowatt hours per day.

(a) If $E$ represents the excess left and $d$ represents the number of days since the heat has been turned on, write an equation for $E$ in terms of $d$.

(b) How much of the excess will be left after one month (use a month length of 30 days)?

(c) If the heat will need to be turned on for 5 months, will the excess be enough to last through this time period? Justify your answer.
3. As Evin is driving her car, she notices that after 1 hour her gas tank has 7.25 gallons left and after 4 hours of
driving, it has 3.5 gallons of gas left in it.

(a) Represent this information as two coordinate pairs in the form \((h, g)\), where \(h\) is the
number of hours driven and \(g\) is the gallons of gas left.

(b) Find the slope between these two points. Using proper units, interpret this slope.

(c) Assuming the relationship between \(h\) and \(g\) is linear, find an equation for \(g\) in terms of \(h\).

(d) According to this equation, after how many hours of driving would Evin run out of gas?

4. The population of Champaign, Illinois is given for three years in the table below:

<table>
<thead>
<tr>
<th>Year</th>
<th>Population</th>
</tr>
</thead>
<tbody>
<tr>
<td>1970</td>
<td>163,488</td>
</tr>
<tr>
<td>1980</td>
<td>168,392</td>
</tr>
<tr>
<td>2012</td>
<td>203,276</td>
</tr>
</tbody>
</table>

(a) Using 1970 as \(t = 0\), create a linear model from the first two data points in this table to
predict the population, \(p\), as a function of the number of years since 1970, \(t\).

(b) If this model is used to predict the population of Champaign in the year 2012, will the model
overestimate or underestimate the actual population? Explain.
Although they don’t fit the classic linear model, it is important to understand how we write equations for horizontal and vertical lines. The first exercise will illustrate the idea. Never forget, though, that when we create an equation for a curve, it simply describes what all points on the curve share in common.

**Exercise #1:** Shown below are a horizontal line and a vertical line.

<table>
<thead>
<tr>
<th>HORIZONTAL LINE</th>
<th>VERTICAL LINE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Write down two coordinate points:</td>
<td>Write down two coordinate points:</td>
</tr>
<tr>
<td>What do they share in common?</td>
<td>What do they share in common?</td>
</tr>
<tr>
<td>What is this line’s equation?</td>
<td>What is this line’s equation?</td>
</tr>
</tbody>
</table>

Equations of horizontal lines and vertical lines are so simple that students will often get them confused later, because they don’t really seem like typical linear equations (because they aren’t).

**HORIZONTAL AND VERTICAL LINES**

**Horizontal Line:** \( y = \text{constant} \)  
**Vertical Line:** \( x = \text{constant} \)

(Constants can be determined by using any point the line passes through)

**Exercise #2:** Which of the following equations represents a vertical line that passes through the point \((5, -3)\)?

1. \( y = -3 \)
2. \( x = 5 \)
3. \( y = -3x + 5 \)
4. \( y = 5x - 3 \)
It is important to be able to quickly and accurately graph vertical and horizontal lines as well as give their equations based on their graphs. We will try to build some fluency with this in the next exercise.

**Exercise #3:** For each of the following, give the equation of the line shown or described.

(a) ![Graph](image1)
   **Equation:** ____________

(b) ![Graph](image2)
   **Equation:** ____________

(c) ![Graph](image3)
   **Equation:** ____________

(d) ![Graph](image4)
   **Equation:** ____________

(e) ![Graph](image5)
   **Equation:** ____________

(f) ![Graph](image6)
   **Equation:** ____________

(g) The equation of a vertical line passing through the point \((-4, 5)\).

(h) The equation of a horizontal line passing through the point \((3, 2)\).

---

**Exercise #4:** Sketch the region bounded by the three lines whose equations are given below. Label each with its equation. Find the area of the triangular region enclosed by the lines. You may want to use your calculator to create a table of values of the first line or simply use facts about the slope and \(y\)-intercept.

\[
y = 2x - 4
\]

\[
x = -1
\]

\[
y = 2
\]
FLUENCY

1. For each of the following, give the equation of the line shown.

(a) \[ \text{Equation: } \] 
(b) \[ \text{Equation: } \] 
(c) \[ \text{Equation: } \] 
(d) \[ \text{Equation: } \] 
(e) \[ \text{Equation: } \] 
(f) \[ \text{Equation: } \]

2. Write the equations of lines that fit the following descriptions. Sketch a picture if needed.
(a) A vertical line that passes through the point \((4, -7)\).
(b) A horizontal line that passes through the point \((-2, 3)\).

(c) A line parallel to the \(x\)-axis that passes through the point \((-2, 15)\).
(d) A line perpendicular to the \(x\)-axis that passes through the point \((5, 1)\).
3. Each of the following lines are either horizontal, vertical, or slanted. Label each with its type and then graph on the grid. Label each with its equation.

(a) \( y = \frac{3}{5}x - 2 \)

(b) \( y = 6 \)

(c) \( y = -x + 7 \)

(d) \( x = -4 \)

(e) \( y = 2x + 1 \)

4. A rectangle is surrounded by the lines whose equations are shown below. Graph these lines and find the area of the rectangle enclosed by them.

\( x = -4 \) \hspace{1cm} \( x = 3 \)

\( y = -2 \) \hspace{1cm} \( y = 2 \)

Area: __________________________

5. The triangular region shown below is bordered by one vertical line, one horizontal line, and one slanted line. State the equation of each line and determine the triangle’s area.

Vertical Line: __________________

Horizontal Line: ________________

Slanted Line: __________________

Area: __________________________
There are two very interesting functions that can be considered related to linear, the **absolute value function** and the **step function**. Let’s start with the simpler of the two, the **absolute value**.

**Exercise #1**: The absolute value gives us the “size” or **magnitude** of a number. Find each of the following.

(a) $|−7|$
(b) $|−2|$
(c) $|6|$
(d) $|0|$

O.k. So, that is easy enough. Now, what does the basic **absolute value** function “look like.”

**Exercise #2**: Consider the absolute value function $f(x) = |x|$. Do the following.

(a) Evaluate $f(−7)$ and $f(4)$.

(b) Fill out the table below and graph the function over this interval. This should be extremely quick.

(c) What is the minimum value of the function on this interval?

(d) Over what domain interval is $f(x) = |x|$ increasing?

**Exercise #3**: For the function $f(x) = |x−4| + 7$ which of the following is the value of $f(1)$? Show the calculations that lead to your answer.

(1) 10
(2) $−2$
(3) 12
(4) 4
Step functions are another type of function that is related to the linear family. It’s graph will reflect its well chosen name.

**Exercise #4:** Consider the step function given by \( f(x) = \begin{cases} 2 & 0 \leq x < 5 \\ 6 & 5 \leq x \leq 10 \end{cases} \).

(a) Evaluate each of the following. After you do your evaluation, write down what coordinate point must lie on the graph as a result of the calculation.

\[
\begin{align*}
 f(0) &= \\
 f(2) &= \\
 f(4) &= \\
 f(5) &= \\
 f(7) &= \\
 f(10) &= 
\end{align*}
\]

(b) Graph the step function on the grid to the right.

Step functions can arise in the real world whenever the output to a particular function is constant over particular ranges. Here’s an example

**Exercise #5:** At a local amusement park, the park charges an admission based on age. Graph the amount of money a person would have to pay for admission based on their age. Remember that someone who is one day short of 4 years old can consider themselves three and under.

<table>
<thead>
<tr>
<th>Age Range</th>
<th>Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>3 and under</td>
<td>Free</td>
</tr>
<tr>
<td>8 and under</td>
<td>$4.00</td>
</tr>
<tr>
<td>16 and under</td>
<td>$8.00</td>
</tr>
<tr>
<td>17 and older</td>
<td>$12.00</td>
</tr>
</tbody>
</table>
ABSOLUTE VALUE AND STEP FUNCTIONS
COMMON CORE ALGEBRA I HOMEWORK

FLUENCY

1. Consider the absolute value function \( f(x) = |x + 3| \) only on the interval \(-6 \leq x \leq 2\).

   (a) Evaluate \( f(-5) \) and \( f(2) \) without a calculator.

   (b) Graph this function over the interval \(-6 \leq x \leq 2\).

   (c) Over which of the following intervals is \( f(x) \) always increasing? Circle the correct choice.

   (1) \(-6 < x < -3\)  \hspace{1cm} (3) \(-4 < x < 0\)
   (2) \(-2 < x < 1\)  \hspace{1cm} (4) \(-5 < x < 2\)

   (d) State the range of \( f(x) \) on this domain interval.

2. Are the two expressions \(|x - 5| \) and \(|x| - 5\) equivalent? Give evidence to support your yes or no answer. Remember, for expressions to be equivalent, they must have the same value for all values of the input variable, \( x \).
3. For each of the following step functions, produce a graph on the grid given.

(a)  
\[ f(x) = \begin{cases} 
-4 & -5 \leq x < 0 \\
4 & 0 \leq x \leq 5 
\end{cases} \]

(b)  
\[ g(x) = \begin{cases} 
10 & 0 \leq x < 4 \\
7 & 4 \leq x < 8 \\
4 & 8 \leq x \leq 12 
\end{cases} \]

APPLICATIONS

4. Postage rates on envelopes are a great example of step functions. There is a fixed price for a certain range of weights and then another fixed price for another range of weights, etcetera. Below is the graph of one such price structure.

(a) According to this graph, what would be the postage rate on a letter weighing 1.5 ounces?

(b) What would be the postage rate on a letter weighing exactly 3.0 ounces?

(c) Write a piecewise defined function for the postage rates:

\[ y' = \begin{cases} 
2.0 & 1.0 \leq x < 2.0 \\
3.0 & 2.0 \leq x < 3.0 \\
4.0 & 3.0 \leq x \leq 4.0 
\end{cases} \]

(d) Why would it be incorrect to state that the range of this function is \( 0.50 \leq y \leq 1.15 \)?
The Truth About Graphs
Common Core Algebra I

At this point we’ve looked at graphs of linear functions and more general functions as simply being plots of input/output pairs. And, for functions, this makes a lot of sense. But, more generally, we want to be able to define points that lie on the graph of an equation or on an inequality with a simple test/definition.

Graphing Equations and Inequalities

The connection between graphs and equations/inequalities is a simple one:

1. Any coordinate pair \((x, y)\) that makes the equation or inequality true lies on the graph.
2. The entire graph is a collection of all of the \((x, y)\) pairs that make the equation or inequality true.

Exercise #1: Consider the linear equation \(y = 4x + 2\).

(a) Does the point \((2, 10)\) lie on the graph of this equation? Justify your answer.

(b) Does the point \((-1, 4)\) lie on the graph of this equation? Justify your answer.

Exercise #2: The equation \(y = 2x^2 - x + 5\) describes a parabola. Does the point \((3, 20)\) lie on its graph? Justify how you found your answer.

Inequalities can also be graphed and we will concentrate on that in the next lesson. But, in this lesson we can certainly determine if particular points will lie on the graph of an inequality.

Exercise #3: Determine for each of the following inequalities whether the point given lies on its graph.

(a) \((4, 1)\) for \(y > 2x - 5\) 
(b) \((2, 8)\) for \(x + y \leq 10\)

(c) \((-3, 2)\) for \(y < x^2 - 4\) 
(d) \((-6, -1)\) for \(y \geq \frac{x + 12}{3}\)
We can even determine, with some additional calculations, whether a point is a solution to a system of equations or a system of inequalities. You’ve studied systems before and we will devote the next unit to them. But, with a simple definition you can “easily” tell whether points are solutions.

### Systems of Equations

A system of equations is a collection of two or more equations joined by the AND truth condition. Because the AND condition is only true when all of its components are true, the solution set of a system is:

The collection of all points that result in all equations or inequalities being true.

That is an extremely important idea. Let’s test it out in the next exercise:

**Exercise #4:** Determine if the point \((3, 1)\) is a solution to the system of equations shown below. Justify your work.

\[
y = 2x - 5 \\
\text{and} \\
y = -4x + 13
\]

Most of the time, the word AND will not be included as it was above. The assumption will be that by telling you that it is a system you know that all of the equations/inequalities are connected with an AND.

**Exercise #5:** Does the point \((5, 15)\) lie in the solution set of the system of inequalities shown below?

\[
y \geq 4x - 7 \\
y < x^2 - 10
\]

You can even mix equations and inequalities because the answer always depends on whether all conditions are true or not.

**Exercise #6:** Is the point \((-2, 5)\) a solution to the system shown below? Justify your answer carefully.

\[
y > \frac{4-x}{2} \\
y = 3x + 11
\]
1. Which of the following points lies on the graph of $y = 3x - 5$?

(1) $(1, -5)$
(2) $(2, 0)$
(3) $(4, 7)$
(4) $(5, 5)$

2. Which of the following points does not lie on the graph of $y = \frac{1}{2}x + 3$?

(1) $(10, 8)$
(2) $(-2, 2)$
(3) $(0, 3)$
(4) $(-6, -3)$

3. Which of the following points would not lie on the line $y = 7$?

(1) $(-2, 7)$
(2) $(7, -1)$
(3) $(0, 7)$
(4) $(5, 7)$

4. For the inequality $y > 4x + 1$ determine if each of the following points does or doesn’t lie in its solution. Show the work that leads to your answer.

(a) $(2, 15)$
(b) $(4, 10)$
(c) $(0, 1)$
(d) $(-3, -8)$
5. Determine if the point \((4, 7)\) is a solution to the system of equations shown below. Justify your yes/no answer.

\[
\begin{align*}
y &= 2x - 1 \\
\text{and} \\
\frac{1}{2}x + 5
\end{align*}
\]

6. One of the following two points lies in the solution set of the system of inequalities below. Determine which point it is and explain why your choice lies in the solution set and the other does not.

\[
\begin{align*}
x + y &< 10 \\
y &\geq \frac{2}{3}x - 2
\end{align*}
\]

\((6, 1)\) \hspace{1cm} \((3, 5)\)

**REASONING**

7. James quickly sketched the graphs of \(y = -4x + 10\) and \(y = 2x + 3\). His graph is shown below. Explain how you know that his graph is inaccurate.

![Graph](image)

8. The point \((4, 20)\) lies on the line \(y = mx + 8\), for some value of \(m\).

(a) If \(m = 2\), will the point \((4, 20)\) lie on the line? How can you tell?

(b) Find the value of \(m\) for which the point \((4, 20)\) will lie on the line.
GRAPHING INEQUALITIES

So, we have graphed linear functions and in the last lesson learned that the points that lie on a graph are simply the \((x, y)\) pairs that make the equation true. Graphing an inequality in the \(xy\)-plane is exactly the same.

Exercise #1: Consider the inequality \(y > x + 3\).

(a) Determine whether each of the following points lies in the solution set (and thus on the graph of) the inequality given.

\[
\begin{array}{ccc}
(2, 7) & (0, 1) & (1, 4)
\end{array}
\]

(b) Graph the line \(y = x + 3\) on the grid below in dashed form. Why are points that lie on this line not part of the solution set of the inequality?

(c) Plot the three points from part (a) and use them to help you shade the proper region of the plane that represents the solution set of the inequality.

(d) Choose a fourth point that lies in the region you shaded and show that it is in the solution set of the inequality.

(e) The point \((10, 12)\) cannot be drawn on the graph grid above, so it is difficult to tell if it falls in the shaded region. Is \((10, 12)\) part of the solution set of this inequality? Show how you arrive at your answer.
There are some challenges to graphing linear inequalities, especially if the output, \( y \), has not been solved for. Let’s look at the worst case scenario.

**Exercise #2:** Consider the inequality \( 3x - 2y \geq 2 \)

(a) Rearrange the left-hand side of this inequality using the commutative property of addition.

(b) Solve this inequality for \( y \) by applying the properties of inequality that we used in Unit #2.

(c) Shade the solution set of this inequality on the graph paper below.

(d) Pick a point in the shaded region and show that it is a solution to the original inequality.

The final type of inequality that we should be able to graph quickly and effectively is one that involves either a horizontal line or a vertical line.

**Exercise #3:** Shade the solution set for each of the following inequalities in the \( xy \)-planes provided. First, state in your own words the \((x, y)\) pairs that the inequality is describing.

(a) \( x < 4 \)  

Your own words:

(b) \( y \geq -2 \)  

Your own words:
GRAPHS OF LINEAR INEQUALITIES
COMMON CORE ALGEBRA I HOMEWORK

FLUENCY

1. Determine which of the following points lie in the solution set of the inequality \( y \geq 2x - 4 \) and which do not. Justify each choice.

   (a) \((5, 4)\)  
   (b) \((0, -1)\)

   (c) \((10, 16)\)  
   (d) \((2, -1)\)

2. Which of the following points lies in the solution set of the inequality \( y \geq 3x + 10 \)?

   (1) \((1, 10)\)  
   (3) \((4, 20)\)

   (2) \((-1, 3)\)  
   (4) \((2, 16)\)

3. Which of the following points does not lie in the solution set to the inequality \( y \geq -\frac{1}{3}x + 5 \)?

   (1) \((6, 3)\)  
   (3) \((-3, 8)\)

   (2) \((-6, 5)\)  
   (4) \((12, 3)\)

4. Which of the following linear inequalities is shown graphed below?

   (1) \(y < \frac{3}{2}x - 1\)  
   (3) \(y > \frac{2}{3}x - 1\)

   (2) \(y \leq \frac{2}{3}x - 1\)  
   (4) \(y \geq \frac{3}{2}x - 1\)
5. Graph the solution set to the inequality shown below. State one point that lies in the solution set and one point that does not.

\[ y < -2x + 4 \]

One Point In Solution: One Point Not In Solution:

6. Rearrange the inequality below so that it is easier to graph and then sketch its solution set on the grid given. Be careful when dividing by a negative and remember to switch the inequality sign.

\[ x - 2y \leq 6 \]

One Point In Solution: One Point Not In Solution:

7. Graph the solution set to each of the following inequalities.

(a) \[ y \leq 4 \]

(b) \[ x > 1 \]
**Introduction to Sequences**  
*Common Core Algebra I*

A sequence is a very special type of function. When students first encounter sequences, they often think of them as just a list of numbers in some particular order (and then they have to find the pattern). We will start with the technical definition of a sequence in terms of a function.

**Sequence Definition**

A sequence is a function whose set of inputs, the domain, is a subset of the natural numbers, i.e. \( \{1, 2, 3, 4, \ldots\} \). A sequence is often shown as an ordered list of numbers, called the terms or elements of the sequence. Sequence function notation can be tricky.

**Exercise #1**: Consider the sequence below. If we represent this sequence with the letter \( a \) then do the following.

\[
4, 8, 16, 32, 64, 128, 256
\]

(a) Find \( a(3) \)  
(b) Find \( a(1) + a(7) \)  
(c) Find \( a_2 \).

(d) Find \( (a_1)^2 \)  
(e) Find \( a_5 - a_4 \)  
(f) Solve for \( n \): \( a(n) = 128 \).

Sequences are functions. The key here is that the input is simply the number’s place in line so to speak and the output is the actual number in the list.

**Exercise #2**: Consider the sequence defined by the formula \( a(n) = 2n + 1 \).

(a) Write out the first 5 elements of this sequence.

(b) Graph the sequence on the grid shown below for \( 1 \leq n \leq 5 \).

(c) Why shouldn’t we connect the points plotted with a continuous straight line?

(d) What is the 21st term of this sequence? Show how you arrived at your answer.
Sequences can be defined by a classic function formula, like what we saw in Exercise #2, and they also can be defined recursively. A recursive formula is one where each term in the sequence depends on a term or terms that came before it.

**Exercise #3:** Consider a sequence of numbers given by the following definition:

\[ b_i = 7 \quad \text{and} \quad b_i = b_{i-1} + 4 \]

(a) Give a common sense interpretation for this recursive function rule.
(b) Write out the rule for the first 4 terms and evaluate each one of them (except \( b_1 \) which is given).

One of the most famous of all recursively defined sequences is known as the Fibonacci Sequence. Let’s play around with it in the next exercise.

**Exercise #4:** The Fibonacci Sequence is defined recursively as follows:

\[ a(1) = 1, \quad a(2) = 1 \quad \text{and} \quad a(n) = a(n-1) + a(n-2) \]

(a) How do you interpret this recursive rule? Write it down in your own words.
(b) Write down the rule for \( a(3), a(4), \) and \( a(5) \) and determine their values.

Sequences often show up in the real world, where they are sometimes defined in terms of a recursive process.

**Exercise #5:** Kirk is trying to train for the marathon. His first month, he runs 5 miles per workout. He adds an additional 3 miles to his workout for each month that he trains.

(a) Fill out the table below for the amount of miles he runs as a function of how many months he has been running.

<table>
<thead>
<tr>
<th>( m )</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a(m) )</td>
<td>5</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(b) Give a recursive definition for the sequence \( a(m) \). Don’t forget to give an initial value.
(c) Graph this sequence for \( 1 \leq m \leq 5 \).
INTRODUCTION TO SEQUENCES
COMMON CORE ALGEBRA I HOMEWORK

FLUENCY

1. Consider the sequence below. If we represent this sequence with the letter \( a \) then do the following.

\[ 1, 7, 13, 19, 25, 31, 37, 43 \]

(a) Find \( a(5) \)

(b) Find \( a_2 + a_6 \)

(c) Find \( a(4) + 2a(6) \)

(d) Find \( \sqrt{a(5)} \)

(e) Find \( \frac{a(5) - a(3)}{2} \)

(f) Find a recursive definition for the sequence \( a(n) \).

2. Consider the sequence defined in the table below.

<table>
<thead>
<tr>
<th>( n )</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>( b(n) )</td>
<td>2</td>
<td>12</td>
<td>22</td>
<td>32</td>
<td>42</td>
</tr>
</tbody>
</table>

(a) Find \( b(4) \)

(b) Find \( \frac{2b(2) - b(3)}{4} \)

(c) Find a recursive definition for the sequence \( b(n) \).
3. Consider a sequence of numbers given by the definition \( c_1 = 2 \) and \( c_i = c_{i-1} \cdot 3 \)

(a) Write out the first 4 terms of this sequence. 
(b) Find the value of \( c_4 - c_2 \). Show your calculation.

APPLICATIONS

4. Erin is traveling abroad this summer and would like to have a bit of spending cash while she’s overseas. She has 100 dollars already saved and she plans on saving 40 dollars a month.

(a) Fill out the table below for the amount of money she saves as a function of how many months she has been saving.

<table>
<thead>
<tr>
<th>( m )</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a(m) )</td>
<td>140</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(b) Give a recursive definition for the sequence \( a(m) \). Don’t forget to give an initial value.

(c) Graph this sequence for \( 1 \leq m \leq 5 \).

REASONING

5. A sequence is defined recursively as follows: \( a(1) = 1 \), and \( a(n) = a(n-1) + n \)

(a) How do you interpret this recursive rule? Write it down in your own words.
(b) Write down the rule for \( a(2), a(3), \) and \( a(4) \) and determine their values.
**ARITHMETIC SEQUENCES**
**COMMON CORE ALGEBRA I**

There are many types of sequences, but there is one that is related to linear functions and in fact is a type of **discrete linear function**. These are known as **arithmetic sequences**. Let’s illustrate one first.

**Exercise #1**: Evin is saving money for to buy a new toy. She already has $12 in her account. She gets an allowance of $4 per week and plans to save $3 in her account.

(a) Fill out the table below for the amount of money Evin has after \( n \) weeks of saving.

<table>
<thead>
<tr>
<th>( n )</th>
<th>( a(n) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$15</td>
</tr>
<tr>
<td>2</td>
<td>$18</td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td></td>
</tr>
</tbody>
</table>

(b) Write a **recursive definition** for this sequence.

(c) What’s wrong with the graph of the sequence shown below?

(d) Evin proposes the following explicit formula for the amount of savings, \( a_n \), as a function of the number of weeks saved \( n \). Is this formula correct? Test it!

\[
a(n) = 3n + 15
\]

**Arithmetic sequences** are ones where the terms in the list increase or decrease by the same amount given a unit increase in the **index** (where the number is in line).

**Exercise #2**: An arithmetic sequence is given using the recursive definition: \( b_1 = -3 \) and \( b_i = b_{i-1} + 6 \). Which of the following is the value of \( b_4 \)? Show the work that leads to your answer.

(1) 24  
(2) 12  
(3) 21  
(4) 15
Arithmetic sequences are relatively easy to spot and are easy to fill in, so to speak.

**Exercise #3:** For each of the following sequences, determine if it is arithmetic based on the information given. If it is arithmetic, fill in the missing blank. If it is not, show why.

(a) 5, 9, 13, ________, 21, 25  
(b) 5, 10, 20, 40, ________, 160

(c) 7, 4, 1, ________, −5, −8  
(d) 64, 16, 4, ________, \(\frac{1}{4}\), \(\frac{1}{16}\)

Finding a specific term in an arithmetic sequence without listing them sometimes can be a challenge, but not if you take your time and really think about it.

**Exercise #4:** Consider an arithmetic sequence whose first three terms are given by: 4, 14, 24

(a) What is the 4th term? How many times was 10 added to 4 to get to the 4th term? Show a diagram to illustrate this.

(b) Use what you learned in part (a) to find the value of \(a_{10}\), the 10th term.

(c) Write a recursive formula for the \(a_n\) based on the term number \(n\).

(d) Write an explicit formula for \(a_n\).

**Exercise #5:** Seats in a small amphitheater follow a pattern where each row has a set number of seats more than the last row. If the first row has 6 seats and the fourth row has 18, how many seats does the last row, which is the 20th, have in it? Show your work to justify your response.
**ARITHMETIC SEQUENCES**

**COMMON CORE ALGEBRA I HOMEWORK**

**FLUENCY**

1. An arithmetic sequence is given using the recursive definition: \( b_1 = 8 \) and \( b_i = b_{i-1} - 2 \). Which of the following is the value of \( b_4 \)? Show the work that leads to your answer.

   (1) 14  
   (2) 2  
   (3) 6  
   (4) 4

2. For each of the following sequences, determine if it is arithmetic based on the information given. If it is arithmetic, fill in the missing blank. If it is not, show why.

   (a) 12, 24, 36, _______, 60, 72  
   (b) 10000, 1000, _______, 10, 1

   (c) _______, 24, 20, 16, 12, 8  
   (d) \( \frac{1}{4} \), \( \frac{1}{2} \), _______, 1, \( \frac{5}{4} \)

3. Given a sequence defined by the explicit formula \( g(n) = 15n + 35 \), write out the first four terms. Then, create a recursive definition and graph the sequence on the interval \( 1 \leq n \leq 7 \).
4. Which of the following is an arithmetic sequence?

(1) 2, 4, 8, 16, 32, 64
(2) 50, 45, 40, 35, 30
(3) 1, 1, 2, 3, 5, 8, 13
(4) 1, \(\frac{1}{2}\), \(\frac{1}{4}\), \(\frac{1}{8}\), \(\frac{1}{16}\)

APPLICATIONS

5. Evin is building a tower out of paper cups. In each row (counting from the floor up), there are two less cups than the row below it. The first row has 26 cups in it.

(a) State the number of cups in the second, third, and fourth rows.

(b) Give a recursive definition for this arithmetic sequence.

(c) How many cups will be in the 11th row? Show the calculation that leads to your answer.

REASONING

6. Eric considers a sequence of numbers given by the following definition \(b_i = 7 + 4 \times i\) and decides the first 4 numbers are:

4, 11, 18, 25

(a) Interpret in your own words, what the sequence is saying and what he actually did.

(b) What should the first four numbers be?
UNIT #5

SYSTEMS OF LINEAR EQUATIONS AND INEQUALITIES

Lesson #1 – Solutions to Systems and Solving by Graphing
Lesson #2 – Solving Systems by Substitution
Lesson #3 – Properties of Systems and Their Solutions
Lesson #4 – The Elimination Method
Lesson #5 – Modeling with Systems of Equations
Lesson #6 – Solving Equations Graphically
Lesson #7 – Solving Systems of Inequalities
Lesson #8 – Modeling with Systems of Inequalities
SOLUTIONS TO LINEAR SYSTEMS AND SOLVING BY GRAPHING
COMMON CORE ALGEBRA I

Systems of equations (and inequalities) are essential to modeling situations with multiple variables and multiple relationships between the variables. At the end of the day, though, the solution set of a system of equations can be easily defined:

SOLUTIONS TO A SYSTEM OF EQUATION

1. A point \((x, y)\) is a solution to a system if it makes all equations true.

2. The solution set of a system is the collection of all pairs \((x, y)\) that are solutions to the system (see 1).

Exercise #1: Determine if the point \((2, 5)\) is a solution to each of the systems provided. Show the work that leads to your answer for each.

(a) \[
y = 4x - 3
2x + y = 9
\]

(b) \[
y - x = 3
y = \frac{1}{2}x + 6
\]

We can solve a system by using a graph. Review this process in the next exercise.

Exercise #2: Consider the system of equations shown below:

\[
\begin{align*}
y & = 2x + 5 \\
y & = 2 - x
\end{align*}
\]

(a) Graph both equations on the grid shown. Use TABLES on your calculator to make the process faster, if necessary. Label each line with its equation.

(b) At what point do the two lines intersect?

(c) Show that this point is a solution to the system.
It’s easy to see why the method of graphing works if you understand the truth about graphs. Remember:

**GRAPHS OF EQUATIONS**

1. A point \((x, y)\) lies on a graph of an equation if it makes that equation **true**.
2. The graph of an equation is simply the set of all points \((x, y)\) that make the equation **true**.

**Exercise #3:** So, now you can put the definition of the graph of an equation together with the definition of a system. Fill in the blanks with one of the words shown:

TRUE, INTERSECTION, SOLUTIONS, BOTH

1. To solve a **system of equations graphically** you find the **intersection** of the two graphs.
2. This works because any intersection point must lie on **both** graphs.
3. Because intersection points lie on both graphs, they must make both equations **true**.
4. Because intersection points make both equations true, they are **solutions** to the system of equations.

We will often use this graphical method to solve systems in applied problems. Let’s take a look at a modeling problem involving a linear system of equations.

**Exercise #4:** Janelle and Swetha are taking a 50 question true false test in their history class. Janelle started after Swetha had already finished 12 questions. Janelle answers questions at a rate of two per minute, while Swetha answers them at a rate of 5 questions every 4 minutes. Janelle eventually catches up to Swetha. How many minutes does it take her and what question are they on when Janelle catches up?

(a) Create two linear models for Janelle and Swetha’s questions answered since Janelle started. It may help to plot some points on the graph paper. Show the work that you use.

(b) Graph the two equations, using your calculator as needed, and solve the problem.
1. Determine whether each of the following points is a solution to the given system. Justify your answer.

(a) \((3, 4)\)
\[\begin{align*}
x + y &= 7 \\
y &= 2x - 2
\end{align*}\]

(b) \((-10, -1)\)
\[\begin{align*}
y &= \frac{1}{2}x + 4 \\
y &= 4x + 30
\end{align*}\]

(c) \((2, 14)\)
\[\begin{align*}
y &= -3x + 20 \\
y &= 2x + 10
\end{align*}\]

(d) \(\left(2, \frac{3}{2}\right)\)
\[\begin{align*}
y &= \frac{8-x}{4} \\
y &= \frac{5}{4}x - 1
\end{align*}\]

2. Solve the following system of equations graphically. After graphing, be sure to label each line with its equation and state your final solution as a coordinate pair.

\[\begin{align*}
y &= \frac{1}{3}x + 1 \\
x + y &= 5
\end{align*}\]
3. Which of the following points solves the system shown below?

(1) $(1, -4)$  (3) $(2, 8)$  \[ y = 5x - 9 \]
(2) $(3, 6)$  (4) $(-3, 18)$  \[ y = -2x + 12 \]

APPLICATIONS

4. Zeke is racing his little brother Niko. They are running a total of 30 yards and Zeke gives Niko a 12 yard head start. Zeke runs 2 yards every second but Niko only runs 1 yard every 2 seconds. If $x$ represents the number of seconds they have been racing and $y$ represents the distance from the start line then:

(a) Fill out the table below for various distances that Zeke and Niko are from the start line at the given times.

<table>
<thead>
<tr>
<th>$x$ (sec)</th>
<th>Zeke Distance (yds)</th>
<th>Niko Distance (yds)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>12</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(b) Based on your calculations for (a) write equations for both Zeke’s distance and Niko’s distance from the start line as a function of the time, $x$.

Zeke’s Distance: ___________________________  Niko’s Distance: ___________________________

(c) Graph both of these equations on the grid above and determine the number of seconds it takes for Zeke to catch up to Niko. How far are they from the finish line at that point?

REASONING

5. The two lines $y = 6x + 15$ and $y = mx - 4$ intersect at $x = -2$.

(a) What is the $y$-coordinate of their intersection point?
(b) What is the value of $m$?
There are a variety of ways that we solve a system of equations. In the last lesson we saw how to solve them graphically. In this lesson we will review and understand the basis for solving them by a method known as substitution. You have seen this technique in Common Core 8th Grade mathematics, but here we will explore it more deeply.

**Exercise #1:** Consider the system given below and its solution $x = 4$ and $y = 1$.

(a) Show that $(4, 1)$ is a solution to the system.

$$2x + y = 9$$
$$y = x - 3$$

(b) Substitute $x - 3$ in for $y$ in the first equation and show that the point $(4, 1)$ is still a solution to this new equation.

(c) Solve the system by finishing the substitution from (b).

Substitution is a very important technique and we want to be very good at it. It boils down to one of the most important properties of equality:

**EQUALS MAY ALWAYS SUBSTITUTE FOR EQUALS**

**Exercise #2:** Solve the following systems of equations by substitution.

(a) $y = 2x + 5$

$y = -3x - 10$

(b) $4x - 2y = 16$

$y = -5x + 13$
The algebra of systems allows us to solve all sorts of problems that almost seem like riddles.

**Exercise #3:** Max and his father Kirk are comparing their ages. They know that the sum of their ages is 52 and that Kirk is seven years older than four times Max’s age.

(a) If Max’s age is represented by \( m \) and Kirk’s age by \( k \), write a system of equations that describes the two relationships from the problem.

(b) Solve the system using substitution to find both of their ages.

**Exercise #4:** Two cell phone plans offer differing text packages. The two plans are outlined below:

Plan A: $5.00 per month charge along with a charge of $0.03 per text.

Plan B: No per month charge, but a charge of $0.10 per text.

Is there a certain number of texts, when the two plans cost the same amount? Determine your answer by setting up a system of equations that model the two plans.

**Exercise #5:** A man and a woman start 380 feet away from each other and walk in a straight line towards each other. If she is walking at a rate of 6 feet per second and he is walking at a rate of 2 feet per second, when will they meet?
SOLVING SYSTEMS BY SUBSTITUTION
COMMON CORE ALGEBRA I HOMEWORK

FLUENCY

1. Solve each of the following system of equations by substitution.
   (a) \( y = x + 8 \)
       \( y = 4x - 1 \)
   (b) \( y = -3x + 5 \)
       \( 2x + y = 6 \)
   (c) \( 4x + 3y = 37 \)
       \( y = x - 4 \)
   (d) \( x - 5y = -49 \)
       \( y = -2x + 1 \)

2. Given the system shown below do the following:
   \( y = \frac{1}{2}x - 2 \)
   \( y = -3x + 5 \)
   (a) Solve this system graphically using the grid shown.
   (b) Solve this system by substitution. Show your work.
APPLICATIONS

3. Water is flowing from Tank #1 to Tank #2 as shown in the picture. Originally, Tank #1 had 1,540 gallons in it and Tank #2 had 236 gallons in it. Water is draining out of Tank #1 at a rate of 6 gallons per minute and, thus, filling Tank #2 up at a rate of 6 gallons per minute.

(a) Write an equation for each tank that models the volume of water, v in gallons, as a function of the number of minutes, m, that the water has been flowing.

Tank #1: ________________________  Tank #2: _____________________________

(b) Find out how long it takes, to the nearest minute, for the two tanks to have the same number of gallons. Will it take longer or shorter than 2 hours? Justify.

4. A rectangle has a perimeter of 42 feet. Its length, L, is three feet more than twice its width, W.

(a) Create an equation in terms of L and W for the perimeter of the rectangle.

(b) Create an equation that relates L and W based on the length being three feet more than twice the width.

(c) Solve the system of equations that you just created by substitution to find the values of the length and width.

REASONING

5. Assuming that \( a \neq c \), find the \( x \)-value of the intersection point of the lines \( y = ax + b \) and \( y = cx + d \).
There is one final way that we will solve systems of equations, but we won’t look at that until the next lesson. Systems are important because they tell us multiple conditions that relate multiple variables or unknowns. In this lesson, we will experiment with systems and what we can do with them and how this affects their solutions.

**Exercise #1:** Consider the system shown to the right and its solution \((1, 5)\).

(a) Show that \(x = 1\) and \(y = 5\) is a solution to the system of equations.

\[
\begin{align*}
4x + 2y &= 14 \\
x - y &= -4
\end{align*}
\]

(b) Find the sum of the two equations. Is the point \((1, 5)\) a solution to this new equation? Justify your yes/no response.

(c) Multiply both sides of the second equation by 2 to get an equivalent equation. Is the point \((1, 5)\) a solution to this new equation? Justify your yes/no response.

(d) Take the equation you found in (c) and add it to the first equation. What happens? How does this allow us to now solve for the variable \(x\)? Do so, what do you find?

(e) Once you know the value of \(x\), how can you find the value of \(y\)?
So, what we see is that a solution to a system of equations remains a solution to that system under a variety of conditions.

**Solutions to Systems Remain Solutions If**

1. Properties of equality are used to rewrite either of the equations.
2. The equations are added or subtracted or any rewrite is added or subtracted.

Let’s play some more with these ideas, but with a new system.

**Exercise #2:** Consider the system shown to the right:

(a) Show that the point \((3, -1)\) is a solution to the system.

\[
\begin{align*}
4x - 3y &= 15 \\
3x + 2y &= 7
\end{align*}
\]

(b) The point \((3, -1)\) will be a solution to the system shown below. How can you determine this without substituting the point in?

\[
\begin{align*}
8x - 6y &= 30 \\
9x + 6y &= 21
\end{align*}
\]

(c) What happens when you add these two equations together? How can this let you solve for \(x\)? Find it and find \(y\).

**Exercise #3:** Solve the system below using the **method of elimination**. Show the steps in your work and show that your answer is in fact a solution to the system.

\[
\begin{align*}
2x + 4y &= 2 \\
6x + 3y &= -3
\end{align*}
\]
PROPERTIES OF SYSTEMS AND THEIR SOLUTIONS
COMMON CORE ALGEBRA I HOMEWORK

FLUENCY

1. The point \((2, 7)\) is a solution to the system of equations given below.

\[
\begin{align*}
3x + 2y &= 20 \\
x - y &= -5
\end{align*}
\]

(a) Show that this point is a solution. 
(b) Add the two equations together and show that \((2, 7)\) is a solution to the result.

(c) Subtract the two equations (be careful) and show that \((2, 7)\) is a solution to the result. 
(d) Multiply both sides of the second equation by 2. Show that \((2, 7)\) is a solution to the result.

2. The point \((4, -2)\) is a solution to the system of equations \(2x + y = 6\). Which of the following equations would it \emph{not} be a solution to?

\[
\begin{align*}
(1) \quad 3x + 6y &= 0 \\
(2) \quad 2x + 10y &= -12 \\
(3) \quad 2x + 2y &= 12 \\
(4) \quad x - 4y &= 12
\end{align*}
\]

3. Which of the following points is a solution to the system?

\[
\begin{align*}
(1) \quad (4, 1) & \quad \quad (3) \quad (-3, 9) \quad \quad x - 2y = -11 \\
(2) \quad (5, 2) & \quad \quad (4) \quad (3, 7) \quad \quad 5x + 2y = 29
\end{align*}
\]
APPLICATIONS

4. A small movie theater sells children’s tickets for $4 each and adult tickets for $10 each for an animated movie. The theater sells a total of $388 in ticket sales.

(a) If \( c \) represents the number of children’s tickets sold and \( a \) represents the number of adult tickets sold, write an equation that models the information shown above.

(b) Show that \( c = 52 \) and \( a = 18 \) is a solution to this equation (not system).

(c) Show that after multiplying both sides of the equation in (a) by 2, \( c = 52 \) and \( a = 18 \) is still a solution to this equation.

(d) How can you interpret multiplying both sides of the equation by 2 in letter (a) in terms of ticket prices and total ticket sales?

REASONING

In the next lesson, we will reinforce solving systems using the **Method of Elimination**. This last question reinforces why and how the method works.

5. Consider the system of equation:

\[
\begin{align*}
    x + 4y &= 13 \\
    3x + 2y &= 19
\end{align*}
\]

(a) Multiply both sides of the second equation by \(-2\). What equation results?

(b) Add the equation from (a) to the first equation. What happens? What can you now solve for?

(c) Now that you know the value of \( x \), how can you find the value of \( y \)? Find it.

(d) What could you have multiplied both sides of the first equation by to **eliminate** the \( x \) instead of the \( y \)?
In previous courses you have seen how to solve systems graphically and how to solve them by substitution. Today’s lesson will build on the previous one and formally introduce the technique of solving a system by elimination. Remember from the last lesson that:

**Solutions to Systems Remain Solutions If**

1. Properties of equality are used to rewrite either of the equations.
2. The equations are added or subtracted or any rewrite is added or subtracted.

**Exercise #1:** Consider the system shown below. Solve the system two ways, by eliminating \( x \) in (a) and eliminating \( y \) in (b).

(a) Eliminate \( x \) to solve

\[
4x + 5y = 12 \\
-2x + y = 8
\]

(b) Eliminate \( y \) to solve

\[
4x + 5y = 12 \\
-2x + y = 8
\]

(c) Show that the point that you found in (a) and (b) is a solution to this system of equations.

**Exercise #2:** Solve the following system of equations by elimination and check that your answer is a solution to this system.

\[
5x - 2y = 10 \\
2x + 7y = 43
\]
There are many applications of solving systems of linear equations by elimination. One of the more interesting ones comes in finding the equation of a line if you know two points that it goes through.

**Exercise #3:** Consider a line that passes through the points \((-2, -11)\) and \((3, 14)\). We want to find its equation in \(y = mx + b\) form.

(a) Substitute both of the known points into \(y = mx + b\) to create a system of two equations with the parameter \(m\) and \(b\).

(b) Solve this system for \(m\) and \(b\) and write the equation of the line.

**Exercise #4:** Find the equation of the linear function, in \(y = mx + b\) form, shown in the table below.

<table>
<thead>
<tr>
<th>(x)</th>
<th>2</th>
<th>6</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>(y)</td>
<td>3</td>
<td>5</td>
<td>9</td>
</tr>
</tbody>
</table>

We will work extensively in the next lesson with modeling real world scenarios with systems and then solving those systems. But, here is a warm-up.

**Exercise #4:** Two numbers have the following properties. The sum of the larger and twice the smaller is equal to 13. Twice their positive difference is equal to eight. What are the two numbers? Play around with modeling this problem using variables. Create careful let statements and equations that translate the information you are given into a system you can solve.
THE METHOD OF ELIMINATION
COMMON CORE ALGEBRA I HOMEWORK

FLUENCY

1. Solve each of the following systems by the Method of Elimination. These two should be relatively easy. Make sure to understand why.

   (a) \( x - y = 7 \) 
     \( x + y = 5 \)

   (b) \( 2x + 5y = 3 \) 
     \( -2x - y = 5 \)

2. Solve each of the following systems by the Method of Elimination. These will be slightly harder than #1 because you will have to alter one of the equations by multiplication.

   (a) \( x - y = 15 \) 
     \( 4x + 2y = 30 \)

   (b) \( 2x + 3y = 17 \) 
     \( 5x + 6y = 32 \)

3. Solve each of the following systems by the Method of Elimination. In each case you will likely want to alter both equations by multiplication.

   (a) \( 2x + 3y = 16 \) 
     \( 5x - 2y = 21 \)

   (b) \( 6x - 7y = 25 \) 
     \( 15x + 3y = 42 \)
4. Which of the following represents the intersection of the lines whose equations are given below?

\[(1) \ (-1, 16) \quad (3) \ (3, 8) \quad y + 2x = 14\]
\[(2) \ (4, 9) \quad (4) \ (0, 7) \quad y - x = 5\]

APPLICATIONS

5. Use the Method of Elimination to find the equation of the line, in \( y = mx + b \) form, that passes through each set of points. Set up a system first, like we did Exercises #3 and 4 from the lesson. Then, solve the system for the slope, \( m \), and the \( y \)-intercept, \( b \).

(a) \((3,10) \) and \((5,18)\)
(b) \((-2,5) \) and \((6, -7)\)

6. Lilly and Rosie are sisters. The sum of their ages is 19 and the positive difference of their ages is 9. Set up a system of equations involving Lilly’s age, \( L \), and Rosie’s age, \( R \), assuming that Lilly is the older child. Solve the system to find their ages.

7. Shana bought sodas and popcorn for the movies. Sodas cost $3 each and popcorn cost $4 per bag. Shana bought 7 things from the concession, all either sodas or bags of popcorn. Shana spent a total of $26. Write a system of equations involving the number of sodas, \( s \), and the bags of popcorn, \( b \). Solve the system to see how many of each Shana bought.
MODELING WITH SYSTEMS OF EQUATIONS
COMMON CORE ALGEBRA I

Many real world scenarios can be modeled using systems of equations. In fact, when we have two quantities that are related and two ways in which those quantities are related, then we can often set up and solve a system.

Exercise #1: Jonathan has nine bills in his wallet that are all either five-dollar bills or ten-dollar bills.

(a) Fill out the following table to see the dependence of the two variables and how they then determine how much money Jonathan has.

| Number of fives, $f$ | Number of tens, $t$ | Amount of Money, $\$\$
|---------------------|---------------------|-----------------------
| 0                   |                     |                       |
| 1                   |                     |                       |
| 2                   |                     |                       |
| 3                   |                     |                       |

(b) If $f$ represents the number of $5$ bills and $t$ represents the number of $10$ bills, then what does the following expression calculate? Explain.

\[5f + 10t\]

(c) If Jonathan has a total of $55, set up a system of equations involving $f$ and $t$ that could be used to determine how many of each bill he has. Solve the system. Remember that he has 9 total bills.

(d) Let’s say that we were told that Jonathan had seven bills that were all 5’s and 20’s and we were also told that he had a total of $120. Set up and solve a system to help evaluate whether we could have been told true information.
There are many different problems that can be modeled with linear systems. Let’s try another one where we use information given to determine **unit prices**.

**Exercise #2:** Samantha went to a concession stand and bought three pretzels and four sodas and paid a total of $11.25 for them. Raza went to the same stand and bought five pretzels and two sodas and paid a total of $8.25.

(a) Could pretzels have cost $1.75 each and sodas $1.50 each? How can you evaluate based on the information given?

(b) Letting $x$ equal the **unit cost** of a pretzel and letting $y$ equal the **unit cost** of a soda, write a system of equations that models the information given.

We can model information given in a geometric form as well. We should feel relatively comfortable working with rectangles and their perimeters. The next question concerns the relationship between the length and width of a rectangle.

**Exercise #3:** A rectangle has a perimeter of 204 feet. It’s length is six feet longer than twice its width. If $L$ stands for the length of the rectangle and $W$ stands for its width, write a system of equations that models the information given in this problem and solve it to find the length and width of this rectangle.
MODELING WITH SYSTEMS OF EQUATIONS
COMMON CORE ALGEBRA I HOMEWORK

APPLICATIONS

1. A local theater is showing an animated movie. They charge $5 per ticket for a child and $12 per ticket for an adult. They sell a total of 342 tickets and make a total of $2550. We want to try to find out how many of each type of ticket they sold. Let $c$ represent the number of children’s tickets sold and $a$ represent the number of adult tickets sold.

(a) Write an equation that represents the fact that 342 total tickets were sold.

(b) Write an equation representing the fact that they made a total of $2550.

(c) Solve the system you created in (a) and (b) by the Method of Elimination.

2. A catering company is setting up tables for a big event that will host 764 people. When they set up the tables they need 2 forks for each child and 5 forks for each adult. The company ordered a total of 2992 forks. Set up a system of equations involving the number of adults, $a$, and the number of children, $c$, and solve to find out how many of each attended the event.

3. Ilida went to Minewaska State Park one day this summer. All of the people at the park were either hiking or bike riding. There were 178 more hikers than bike riders. If there were a total of 676 people at the park, how many were hiking and how many were riding their bikes?
4. Juanita and Keenan own a camping supply store and just put in an order for flashlights and sleeping bags. The number of flashlights ordered was five times the number of sleeping bags. The flashlights cost $12 each and the sleeping bags cost $45 each. If the total cost for the flashlights and sleeping bags was $1785, how many flashlights and how many sleeping bags did Juanita and Keenan order?

5. For a concert, there were 206 more tickets sold at the door than were sold in advance. The tickets sold at the door cost $10 and the tickets sold in advance cost $6. The total amount of sales for both types of tickets was $6828. How many of each type of ticket was sold?

6. Eldora and Finn went to an office supply store together. Eldora bought 15 boxes of paper clips and 7 packages of index cards for a total cost of $55.40. Finn bought 12 boxes of paper clips and 10 packages of index cards for a total cost of $61.70. Find the cost of one box of paper clips and the cost of one package of index cards.
SOLVING EQUATIONS GRAPHICALLY
COMMON CORE ALGEBRA I

As we have mentioned, there are many different ways to solve equations, i.e. find the value(s) of the variable(s) that result in the equation being true. One of the algebraic methods that we’ve seen is to use inverse operations to undo what has been done to the variable. Other sub-methods involve writing equivalent expressions and manipulating the equation with the properties of equality. Today we will see how to solve equations by using graphs (primarily created on our calculator).

Exercise #1: Consider the equation $3x - 2 = 10 - x$.

(a) Solve this equation using standard methods and then show it is a solution by checking.

(b) Using your calculator, fill out the table below for the two expressions. Circle the solution you found in (a).

<table>
<thead>
<tr>
<th></th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$3x - 2$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$10 - x$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(c) Using your calculator, sketch a graph of the two lines $y = 3x - 2$ and $y = 10 - x$ on the axes below. Use a WINDOW of $-2 \leq x \leq 6$ and $-2 \leq y \leq 11$. Use your calculator to find their intersection point.

(d) Fill in the blanks:

To solve the equation $f(x) = g(x)$ you ______________ both $f(x)$ and $g(x)$ and then find the ______________ point(s) of their graphs.

This is one of, if not the most important, equation solving technique. It literally works for all equations, although will not always result in exact answers. We can even solve equations that we don’t yet have algebraic techniques for.

Exercise #2: Consider the equation $x^2 - 11 = -2x + 4$. We will eventually have (multiple) algebraic methods for solving this quadratic equation. But for now, we will solve it graphically. Solve this equation graphically using a WINDOW of $-8 \leq x \leq 8$ and $-20 \leq y \leq 20$. Show that your answers are solutions by checking the equation.
It is important that we be able to take a graph and use it to help us solve an equation. Take a look at the next exercise which works with an absolute value equation.

**Exercise #3:** The function \( f(x) = 2|x + 1| - 8 \) is shown on the grid below. Use it to help answer the following questions.

(a) Find all value(s) of \( x \) that solve the equation shown below. Circle points on the graph that illustrate your answer.

\[ 2|x + 1| - 8 = 0 \]

(b) Find all value(s) of \( x \) that solve the equation shown below. Illustrate your work on the graph.

\[ 2|x + 1| - 8 = -2 \]

(c) Find all value(s) of \( x \) that solve the equation shown below. This will necessitate a bit more work on your part. Verify that the values of \( x \) that you found are in fact solutions to this equation. Show the steps in your check.

\[ 2|x + 1| - 8 = x - 1 \]

**Exercise #4:** The functions \( y = -x^2 + 4x \) and \( y = 4 - x \) are graphed on the grid shown. Which of the following sets gives all solutions to the equation \(-x^2 + 4x = 4 - x\)?

(1) \( \{0, 3\} \)  
(2) \( \{1, 4\} \)
(3) \( \{1, 3\} \)  
(4) \( \{0, 4\} \)
SOLVING EQUATIONS GRAPHICALLY
COMMON CORE ALGEBRA I HOMEWORK

FLUENCY

1. The functions \( y = x^2 - 2x - 3 \) and \( y = x - 1 \) are graphed on the grid shown below. Which of the following is the solution set of the equation:
\[ x^2 - 2x - 3 = x - 1 \]
(1) \( \{-2, 1\} \)  
(2) \( \{-3, 1\} \)  
(3) \( \{-3, 0\} \)  
(4) \( \{-1, 4\} \)

2. If the quadratic function \( y = -2(x+1)^2 + 8 \) is shown graphed below, then which of the following represents the solutions to:
\[ -2(x+1)^2 + 8 = 0 \]
(1) \( x = 0, 3 \)  
(2) \( x = -4, 8 \)  
(3) \( x = -2, 2 \)  
(4) \( x = -3, 1 \)

3. The quadratic function \( f(x) = x^2 + 2x - 8 \) is shown graphed on the grid below.

(a) What values of \( x \) solve the equation \( x^2 + 2x - 8 = 0 \) based on this graph?

(b) Graph the line \( g(x) = 2x+1 \) on the grid.

(c) What values of \( x \) solve the equation:
\[ x^2 + 2x - 8 = 2x + 1 \]
4. For each of the following equations, use your calculator to solve by graphing both sides of the equation and finding the \( x \)-coordinate(s) of intersection as done in Exercise #2. Use the window indicated by the particular a and sketch a graph to illustrate your answers.

(a) \(-\frac{1}{2}x + 6 = 4x - 3\)

(b) \(x^2 - 7x - 7 = -3x + 5\)

(c) \(x^2 - 8 = |2x|\)

(d) \(x^2 + 4x - 6 = -x^2 + 5x + 4\)

Solution(s): ________________________  Solution(s): ________________

Solution(s): ________________________  Solution(s): ________________

REASONING

5. The graphs of two functions, \( f(x) \) and \( g(x) \), intersect only twice. Selected values of the functions are shown in the table below. Based on the table, state the solutions to the equation:

\[ f(x) = g(x) \]

<table>
<thead>
<tr>
<th>( x )</th>
<th>(-2)</th>
<th>(-1)</th>
<th>(0)</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x) )</td>
<td>7</td>
<td>3</td>
<td>(-2)</td>
<td>(-8)</td>
<td>0</td>
<td>5</td>
</tr>
<tr>
<td>( g(x) )</td>
<td>(-8)</td>
<td>3</td>
<td>5</td>
<td>1</td>
<td>0</td>
<td>(-3)</td>
</tr>
</tbody>
</table>

Solutions: ________________________
We can have systems of inequalities as well as systems of equations (equalities). The definition of solving a system still holds: we have to find all points that make all inequalities true.

**Exercise #1:** Consider the system of inequalities shown below. Determine if each of the following points is a solution or not to the system. Show work that justifies your answers.

(a) \((3, 8)\)  
(b) \((5, 9)\)

(c) Graph the solution set to this system of inequalities.

**Exercise #2:** On the grid shown below, graph the solution to the system of inequalities shown below. State a point that lies in the solution set and one that doesn’t.

\[
y < \frac{3}{2}x + 2 \\
x \geq -2
\]
**Exercise #3:** Which of the following points is a solution to the system of inequalities shown below? Show the work that leads to your answer.

(1) \((3, -6)\) \quad (3) \((-2, 10)\) \quad \(y \leq -4x + 2\)

(2) \((0, 2)\) \quad (4) \((4, 10)\) \quad \(y > \frac{x}{2} + 7\)

Very often, systems of inequalities will define portions of the \(xy\)-plane that can be visualized and manipulated.

**Exercise #4:** Consider the system of inequalities given below.

(a) Determine which, if any, of these points is a solution to the system.

\((-1, 4)\) \quad (3, 1) \quad \(y \geq -2\)

\(x < 4\) \quad \(y \leq 2x\)

(c) Find the area of the portion of the \(xy\)-plane that represents the solution.

(b) Sketch the solution to the system on the grid provided.

(d) Why does the dashed line of one of the borders **not** make a difference in terms of the area you found in part (c)?
SOLVING SYSTEMS OF INEQUALITIES
COMMON CORE ALGEBRA I HOMEWORK

FLUENCY

1. Which of the following points is a solution to the system of inequalities shown below?

   (1) (3, 5)  
   (2) (1, 3)  
   (3) (1, -2) 
   (4) (2, 3) 

   \[ y > x + 1 \]
   \[ y \leq -2x + 7 \]

2. A system of inequalities is shown graphed below. Which of the following points lies in the solution set of this system?

   (1) (-1, 2)  
   (2) (1, 5)  
   (3) (2, -4) 
   (4) (4, 2) 

3. Consider the system of inequalities shown below.

   \[ y > \frac{2}{3}x - 2 \]
   \[ y \leq -x + 6 \]

   (a) Is the origin, (0, 0), part of the solution set of the system? Determine without first graphing.

   (b) Graph the solution to the system of inequalities. Then, state one point that lies in the set and one that doesn’t.

One Point That Lies in the Solution: 
One Point that Does Not Lie in the Solution:

[Diagram of solution set and graphed lines]
4. Sketch the solution to the system of inequalities shown below:

\[ y + 2x < 6 \]
\[ x \leq 2 \]

State a point that lies in the solution set:

5. Find the area of the triangular region defined by the system of inequalities shown below.

\[ y \geq x \]
\[ x \geq -3 \]
\[ y \leq 6 \]

**REASONING**

6. Consider the system of inequalities shown below:

\[ y \geq x + 2 \]
\[ y \leq x - 3 \]

(a) Graph the system solution to the system on the grid.

(b) Why can you **not** state a point in the solution set?
There are many situations that arise in business and engineering that necessitate systems of linear inequalities. The region in the $xy$-plane that solves the systems often represents all of the viable solutions to the system, so being able to visualize this region can be extremely helpful. As always, with modeling, it is important to really read the problems and understand the physical quantities involved.

**Exercise #1:** John mows yards for his father’s landscaping business for $10 per hour and also works at a bakery for $15 per hour. He can work at most 52 hours per week during the summer. He needs to make at least $600 per week to cover his living expenses.

(a) If John works 14 hours mowing and 30 hours at the bakery, does this satisfy all of the problem’s constraints?

(b) If $x$ represents the hours John spends mowing and $y$ represents the hours he spends at the bakery, write a system of inequalities that describes this scenario.

(c) If John must work a minimum of 10 hours for his father, will he be able to make enough money to cover his living expenses? Show the work that leads to your answer.

(d) Graph the system of inequalities with the help of your calculator (if needed) on the axes below. Use the space below to think about how to graph these lines.

(e) John’s father needs him to work a lot at the landscaping business. Show the point on the graph that corresponds to the greatest number of hours that he can work while still covering his expenses.

(f) Algebraically, find the greatest number of hours that John can work for his father and still cover his expenses. Explain how you found your answer or show your algebra below.
**Exercise #2:** For each of the following, write a system of inequalities that models the problem. You do not need to solve the system.

(a) Frank is putting together a bouquet of roses and daisies. He wants at least one rose and at least two more daisies than roses. Roses cost $4 each and daisies cost $2 each. Frank must spend $40 or less on this bouquet. If \( r \) represents the number of roses he buys and \( d \) represent the number of daisies, write the system.

(b) A diet food company is attempting to create a non-carb brownie composed entirely of fat and protein. The brownie must weigh at least 10 grams but have no more than 100 calories. Fat has 9 calories per gram and protein has 4 calories per gram. If \( x \) represent the weight, in grams, of protein and \( y \) represents the weight, in grams, of fat, write the system.

**Exercise #3:** The drama club at a local high school is trying to raise money by putting on a play. They have only 500 seats in the auditorium that they are using and are selling tickets for these seats at $5 per child’s ticket and $10 per adult ticket. They must sell at least $2000 worth of tickets to cover their expenses.

(a) If \( x \) represents the number of children’s tickets sold and \( y \) represents the number of adult tickets sold, write a system of inequalities that models this situation.

(b) Using technology, sketch the region in the coordinate plane that represents solutions to this system of inequalities.

(c) If the students want to sell exactly 500 tickets and make exactly $2000, how many of each ticket should they sell? Why is this answer not realistic?
COMMON CORE ALGEBRA I, UNIT #5 – SYSTEMS OF LINEAR EQUATIONS AND INEQUALITIES – LESSON #8

MODELING WITH SYSTEMS OF INEQUALITIES
COMMON CORE ALGEBRA I HOMEWORK

APPLICATIONS

1. Jody is working two jobs, one as a carpenter and one as a website designer. He can work at most 50 hours per week and makes $35 per hour as a carpenter and $75 an hour as a website designer. He wants to make at least $2350 per week but also wants to work at least 10 hours per week as a carpenter. Let \( c \) represent the hours he works as a carpenter and let \( w \) represent the hours he works as a website designer.

(a) Write a system of inequalities that models this scenario.

(b) What is the maximum amount of money that Jody can make in a week given the system in (a)? Explain your reasoning.

(c) The graph of the system is shown below with its solutions shown shaded. Three lines are graphed. Label each with its equation.

(d) Find the coordinates of point \( A \) by solving a system of equations by Elimination.

(e) What does the value of \( c \) that you found in the solution to part (d) represent about the number of hours Jody can work as a carpenter. Explain your thinking.
2. For each of the following, create a system of inequalities that models the scenarios presented. You do not need to solve the systems.

(a) Two pumps at a local water facility can only run individually. They will run for at least 18 hours in a day but obviously no more than 24 hours in a day. Pump 1 can move 120 gallons per hour while Pump 2 can move 200 gallons per hour. In total the two pumps must move at least 3,000 gallons of water per day. If \( x \) represents the number of hours that Pump 1 runs and \( y \) represents the number of hours that Pump 2 runs, write a system of inequalities that models all conditions.

(b) Dave is buying popcorn and sodas for his son and his three friends that he brings to the movies (four kids total). He needs to buy at least one of the two items for each of the four. Popcorn costs $2.50 per bag and sodas cost $4.00 each. Dave can spend at most $20. If \( s \) represents the number of sodas he buys and \( p \) represents the number of bags of popcorn, then write a system that models this scenario.

REASONING

3. Systems of inequalities can also come in discrete versions where the two variables involved can only take on integer values. Let’s look at a simple example of this.

Jennifer is putting together a selection of flowers that has at most 12 flowers in it. She is choosing either roses or carnations. She wants to pick at least three roses and at least two carnations. Let \( r \) be the number of roses she uses and let \( c \) be the number of carnations she uses.

(a) Write a system of inequalities that models this scenario.

(b) If Jennifer used the minimum number of carnations, what is the maximum number of roses she could use?

(c) What is the fewest flowers Jennifer will use and in what combination?

(d) Graph the solution set to the system. Be careful, this should be a collection of points, not a shaded region.
UNIT #6

EXPONENTS, EXPONENTS, AND MORE EXPONENTS

Lesson #1 – Simplifying Expressions Involving Exponents
Lesson #2 – Zero and Negative Exponents
Lesson #3 – Exponential Growth
Lesson #4 – Introduction to Exponential Functions
Lesson #5 – Percent Review
Lesson #6 – Percent Increase and Decrease
Lesson #7 – Exponential Models Based on Percent Growth
Lesson #8 – Linear Versus Exponential
Lesson #9 – Geometric Sequences
Simplifying Expressions Involving Exponents
Common Core Algebra I

There are many situations in science, engineering and other fields where a process is governed by repeatedly multiplying (or dividing) by the same quantity. Repeated multiplication (and division) is represented by exponents. We have worked with these already, but let’s review some basics in the first exercise.

Exercise #1: Each of the following problems involves basic exponent ideas. Answer each to review your previous knowledge.

(a) Represent $6^3$ as an extended product. Do not evaluate the product.
(b) If $f(x) = 2x^3 + 7$, then $f(-1) =$?
(c) If $x^3 \cdot x^5$ is written in the form of $x^n$ what is the value of $n$? Write extended products if you don’t remember the Exponent Rule.
(d) If the expression $(5x^3)^2$ is written in the form $ax^b$, what is the value of $a + b$?
(e) If the length of a rectangle is $3 \times 10^5$ meters and its width is $2 \times 10^4$ meters, what is its area written in scientific notation?
(f) Rewrite the product $(3x^2)(2x^5)^3$ as an equivalent expression in simplest exponential form.

We also would like to be able to write simpler equivalent expressions involving ratios (or division problems) involving exponents. This all comes down to your ability to “unmultiply” fractions. The next exercise will illustrate.

Exercise #2: Consider the expression $\frac{2x^6}{4x^3}$.

(a) Write this expression as the product of two fractions, one of which is equal to 1.
(b) Simplify the expression.
Let’s see if we can develop a sense on how to simply these types of expressions more quickly.

**Exercise #3:** Rewrite each expression as the product of two fractions, one of which is equal to 1. Then, write it as an equivalent, but simpler, expression.

(a) \( \frac{5^7}{5^3} \)  
(b) \( \frac{x^4}{x^{10}} \)  
(c) \( \frac{x^4 y^8}{x y^{10}} \)

Now, let’s simplify some more complicated exponential expressions. Each time, go back to rewriting the expressions based on basic principles like repeated multiplication and fractions equivalent to 1.

**Exercise #4:** Rewrite each of the following as equivalent exponential expressions in simplified exponential form.

(a) \( \frac{(3x^2)^3}{9x^4} \)  
(b) \( \frac{(5x^2 y^3)^2}{(10xy)^2} \)

**Exercise #5:** The diagram below show how the expression \( \frac{(2x)^2}{(4x)^3} \) gets simplified. For each transition, given the reason (rule, property, etcetera) that justifies the manipulation.

\[
\frac{(2x)^2}{(4x)^3} \rightarrow \frac{2x \cdot 2x}{4x \cdot 4x \cdot 4x} \rightarrow \frac{(2 \cdot 2) \cdot (x \cdot x)}{4 \cdot (x \cdot x) \cdot (4 \cdot 4 \cdot x)} \rightarrow \frac{4x^2 \cdot 1}{4x^2 \cdot 16x} \rightarrow \frac{1}{16x}
\]
SIMPLIFYING EXPRESSIONS INVOLVING EXPONENTS
COMMON CORE ALGEBRA I HOMEWORK

FLUENCY

1. Which of the following is equivalent to \((3x^2y)(10x^5y^3)\)?

(1) \(30x^{10}y^3\)
(2) \(30x^7y^4\)
(3) \(13x^7y^4\)
(4) \(13x^{10}y^3\)

2. If the expression \((2x^4)^3\) was written in \(ax^b\) form, which of the following would be the sum of \(a\) and \(b\)?

(1) 20
(2) 14
(3) 9
(4) 18

3. A square field has a side length of \(6 \times 10^3\) meters. Which of the following is its area in square meters?

(1) \(6 \times 10^6\)
(2) \(36 \times 10^9\)
(3) \(36 \times 10^6\)
(4) \(6 \times 10^9\)

4. Circle the reason for each of the following manipulations used to simplify the product \((8x^2)(3x^3)\).

\[
\begin{align*}
8x^2 \cdot 3x^7 & \quad \Rightarrow \quad 8 \cdot 3 \cdot x^2 \cdot x^7 \\
\text{commutative or associative} & \quad \Rightarrow \quad \text{commutative or associative} \\
\text{commutative or exponent property} & \quad \Rightarrow \quad 24x^9
\end{align*}
\]

5. Rewrite each expression as the product of two fractions, one of which is equal to 1. Then, write it as an equivalent, but simpler, expression.

(a) \(\frac{10^6}{10^2}\)
(b) \(\frac{x^2}{x^6}\)
(c) \(\frac{x^4y}{xy^8}\)
6. Write each of the following expressions equivalently in simplest form.

(a) \( \frac{4x^7}{8x^3} \)   
(b) \( \frac{15x^{10}}{10x^2} \)   
(c) \( \frac{16x}{20x^3} \)   

(d) \( \frac{x^2y^5}{xy} \)   
(e) \( \frac{18x^4y^2}{3x^8y^5} \)   
(f) \( \frac{6x^5y^2}{8xy^3} \)

7. For each of the following fractions, first simplify the numerator and denominator, then simplify the overall fraction. The first is done as an example.

(a) \( \left( \frac{2x^2}{4x} \right)^3 \)   
(b) \( \left( \frac{10x^4}{5x^2} \right)^3 \)   
(c) \( \left( \frac{6x}{4x^2} \right)^3 \)

\[ \frac{8x^6}{16x^2} = \frac{x^4}{2} \]

(d) \( \left( \frac{x^3y^5}{xy^2} \right)^3 \)   
(e) \( \left( \frac{2xy^2}{x^2y^3} \right)^2 \)   
(f) \( \left( \frac{9xy}{3x} \right)^2 \)

REASONING

8. Kris has incorrectly simplified the expression \( \frac{20x^6}{4x^2} \) as \( 5x^3 \).

(a) Show using the value \( x = 2 \) that \( \frac{20x^6}{4x^2} \) and \( 5x^3 \) are not equivalent.   
(b) What is the correct simplification?
In math, people often invent ways to extend concepts to areas that might not make sense at first. Pretty much everyone can understand what \( 2^3 \) means, because they understand that it represents multiplying the number 2, 3 times. Yet, what does \( 2^0 \) or \( 2^{-4} \) mean? Does it make sense to talk about multiplying by a number a negative amount of times? Let’s explore these ideas in the first exercise.

**Exercise #1:** We can think of powers of 2 as representing multiplication of the number 1 repeatedly.

(a) Fill in the pattern for powers that are not negative. What does this lead you to fill in for \( 2^0 \)?

\[
\begin{align*}
2^4 &= \\
2^3 &= \\
2^2 &= 1 \cdot 2 \cdot 2 = 4 \\
2^1 &= 1 \cdot 2 = 2 \\
2^0 &= 
\end{align*}
\]

(b) If **positive exponents** indicated multiplying the number 1 by 2 repeatedly, then **negative exponents** should indicate ______________.

\[
\begin{align*}
2^{-1} &= \frac{1}{2} \\
2^{-2} &= \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4} \\
2^{-3} &= \\
2^{-4} &= 
\end{align*}
\]

We want the pattern of positive, integer powers to extend to zero exponents and negative, integer exponents. We can now define zero and negative exponents as follows.

<table>
<thead>
<tr>
<th>ZERO AND NEGATIVE EXPONENTS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Zero Exponents: ( b^0 = 1 ) as long as ( b \neq 0 ).</td>
</tr>
<tr>
<td>2. Negative Exponents: ( b^{-n} = \frac{1}{b^n} )</td>
</tr>
</tbody>
</table>

**Exercise #2:** Which of the following is not equivalent to \( 5^{-2} \)?

(1) \( \frac{1}{5^2} \)  
(2) \( \frac{1}{10} \)  
(3) \( \frac{1}{25} \)  
(4) 0.04

**Exercise #3:** If \( f(x) = 3x^{-2} + 2x^0 \), then which of the following is the value of \( f(2) \)? Show the work that leads to your answer. Remember, exponents always come before multiplication.

(1) \( 2\frac{1}{4} \)  
(2) \( 1\frac{1}{2} \)  
(3) \( 1\frac{1}{2} \)  
(4) \( 2\frac{1}{2} \)
Because we now have negative exponents we can develop a third exponent law. Recall that we already have the following two.

### EXponent Laws (So Far)
1. \( x^a \cdot x^b = x^{a+b} \)
2. \( (x^a)^b = x^{ab} \)

Now, let’s see if we can develop a rule for dividing quantities that have the same base.

**Exercise #4:** Rewrite each of the following expressions in simplest exponential form.

(a) \( \frac{x^5}{x^2} \)
(b) \( \frac{3^{10}}{3^5} \)
(c) \( \frac{x^8}{x^2} \)

(d) So it appears that: \( \frac{x^a}{x^b} = \)

Now we have a pattern that works quite well if the exponent in the numerator is greater than that of the denominator. But does it work if that isn’t true?

**Exercise #5:** Rewrite each of the following expressions two ways: (i) by using the exponent rule developed in #4(d) and (ii) by simplifying using techniques we have seen in the last lesson.

(a) \( \frac{2^4}{2^7} \)
(b) \( \frac{x^2}{x^7} \)
(c) \( \frac{5^6}{5^{10}} \)

So, we now we see that the subtraction rule for exponents is consistent with negative and zero exponents. For now, we just want to be comfortable that negative exponents indicate division and positive exponents indicate multiplication.

**Exercise #6:** Consider the exponential function \( f(x) = 16(2)^x \). Find each of the following without your calculator.

(a) \( f(0) \)
(b) \( f(2) \)
(c) \( f(-2) \)
ZERO AND NEGATIVE EXPONENTS
COMMON CORE ALGEBRA I HOMEWORK

FLUENCY

1. Rewrite each of the following as equivalent expressions without the use of negative or zero exponents. Remember your order of operations.
   (a) $5^{-3}$
   (b) $6^0$
   (c) $2^{-5}$
   (d) $4x^0$
   (e) $(4x)^0$
   (f) $x^{-2}y^4$

2. Which of the following is not equivalent to $2^{-3}$?
   (1) $\frac{1}{2^3}$
   (2) $-6$
   (3) $0.125$
   (4) $\frac{1}{8}$

3. If $f(x) = 12(2)^x$, then which of the following represents the value of $f(-2)$?
   (1) $-48$
   (2) $6$
   (3) $3$
   (4) $-4$

4. If the expression $8(x+11)^0 - 2x^0 + 6x$ is evaluated when $x = -1$, the result would be
   (1) $1$
   (2) $0$
   (3) $7$
   (4) $4$

5. The numerical expression $\frac{(5^3)^2}{(5^7)^x}$ is equivalent to
   (1) $\frac{1}{25}$
   (2) $25$
   (3) $10$
   (4) $-\frac{1}{10}$
6. Write each of the following in the form \(ax^n\), where \(n\) can be either a positive or negative integer.

(a) \(\frac{x^3}{x^8}\)  
(b) \(\frac{6x}{2x^8}\)  
(c) \(\frac{28x^6}{21x^3}\)

APPLICATIONS

7. The number of people, \(n\), who know a rumor can be modeled using the equation \(n(d) = 20(2)^d\), where \(d\) is the number of days since Monday.

(a) Explain why \(n(0) = 20\). What does this represent in terms of the situation modeled?  
(b) What is the value of \(n(-2)\)? What does this represent in terms of the situation modeled?

REASONING

8. The expression \(\frac{(x^{2a+1})^3}{(x^{a+3})^2}\) can be written as \(x^n\), where \(n\) depends on the value of \(a\).

(a) If \(a = 5\), then find the value of \(n\). Show your work.  
(b) Find a binomial expression for \(n\) in general terms of \(a\).

9. Consider the function \(f(x) = 18(3)^{-x}\). When the value of \(x\) is increased by 1, the output is

(1) multiplied by 3  
(2) divided by 3  
(3) multiplied by \(-3\)  
(4) divided by \(-3\)
There are many things in the real world that grow faster as they grow larger or decrease slower as they get smaller. These types of phenomena, loosely speaking, are known as exponential growth (and decay in the case of decreasing). In today’s lesson, we will look at both growth and decay.

Exercise #1: The number of people who have heard a rumor often grows exponentially. Consider a rumor that starts with 3 people and where the number of people who have heard it doubles each day that it spreads.

(a) Why does it make sense that the number of people who have heard a rumor would grow exponentially?

(b) Fill in the table below for the number of people, \( N \), who knew the rumor after it has spread a certain number of days, \( d \).

<table>
<thead>
<tr>
<th>( d )</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>( N )</td>
<td>3</td>
<td>6</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Exercise #2: We’d like to determine the number of people who know the rumor after 20 days, but to do that, we need to develop a formula to predict \( N \) (the number knowing the rumor) if we know \( d \) (the number of days it has been spreading).

(a) For the following number of days, fill in how you calculated your values based on extended products using the number 2.

\[
\begin{align*}
  d = 0 & \quad N = 3 \\
  d = 1 & \quad N = 3 \cdot 2 \\
  d = 2 & \quad N = (3 \cdot 2) \cdot 2 = 3 \cdot 2 \cdot 2 \\
  d = 3 & \quad N = \\
  d = 4 & \quad N = \\
  d = 5 & \quad N = 
\end{align*}
\]

(b) Using the pattern you developed in (a), write a formula giving the number of people who know the rumor, \( N \), if you know the number of days, \( d \), it has been spreading.

(c) How many people would know the rumor after 20 days?

(d) Exponential growth can be very fast. Assuming our equation from (b) holds, how many days will it take for the number of people knowing the rumor to surpass the population of the United States, which is approximately 315 million people? Show calculations that support your answer.
Let’s now look at developing a fairly simple exponential decay problem.

**Exercise #2:** Helmut (from Finland) is heading towards a lighthouse in a very peculiar way. He starts 160 feet from the lighthouse. On his first trip he walks half the distance to the lighthouse. On his next trip he walks half of what is left. On each consecutive trip he walks half of the distance he has left. We are going to model the distance, \( D \), that Helmut has remaining to the lighthouse after \( n \)-trips.

(a) Fill in the table below for the amount of distance that Helmut has left after \( n \)-trips.

<table>
<thead>
<tr>
<th>( n )</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>( D ) (ft)</td>
<td>160</td>
<td>80</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(b) Each entry in the table could be found by multiplying the previous by what number? This is important because we always want to think about exponential functions in terms of multiplying.

(c) Like in Exercise #2(a), we want to see this process as repeated multiplication by \( \frac{1}{2} \). Fill out each of the following pattern:

\[
\begin{align*}
 n = 0 & \quad D = 160 \\
 n = 1 & \quad D = 160 \cdot \frac{1}{2} = 80 \\
 n = 2 & \quad D = 80 \cdot \frac{1}{2} = \left( 160 \cdot \frac{1}{2} \right) \cdot \frac{1}{2} = \\
 n = 3 & \quad D = \\
 n = 4 & \quad D =
\end{align*}
\]

(d) Based on (c), give a formula that predicts the distance, \( D \), that Helmut has left after \( n \)-trips.

(e) How far is Helmut from the windmill after 6 trips? Provide a calculation that justifies your answer and don’t forget those units!

(f) Helmut believes he will reach the windmill after 10 trips. Is he correct?

(g) Explain why Helmut will never reach the windmill?

(h) Why is the domain of this function only the whole numbers, i.e. \( \{0, 1, 2, 3, \ldots\} \)?
EXPONENTIAL GROWTH AND DECAY
COMMON CORE ALGEBRA I HOMEWORK

APPLICATIONS

1. A piece of paper is 0.01 centimeters (cm) thick. When you fold it once, it becomes 0.02 centimeters thick. If you fold it again, it doubles again to 0.04 centimeters thick. Each fold doubles the thickness of the paper.

(a) How thick is the paper after:

4 Folds:

5 Folds:

(b) For each of the following number of folds, \( f \), show how you can calculate the thickness, \( T \), based on repeatedly multiplying by 2.

\[
\begin{align*}
  f = 0 & \quad T = 0.01 \\
  f = 1 & \quad T = 0.01(2)^1 = 0.02 \\
  f = 2 & \quad T = 0.02(2) = 0.01(2)(2) = 0.01(2)^2 \\
  f = 3 & \quad T = \\
  f = 4 & \quad T =
\end{align*}
\]

(c) Determine a formula, based on (b), for the thickness, \( T \), based on the number of folds, \( f \).

(d) How thick would the paper be if \( f = 10 \)? Use proper units

(e) If there are 100 centimeters in a meter, how many meters thick is the paper after 20 folds? Show the work that leads to your answer.

(d) If there are 1000 meters in a kilometer and the Moon is 384,000 kilometers away from the Earth, will the paper reach the Moon after 40 folds? Show the calculations that lead to your answer.
2. The Sierpinski Triangle is a type of progression where an equilateral triangle has \( \frac{1}{4} \) of its area removed to create a new shape. Then \( \frac{1}{4} \) of its remaining area is taken away. A series of these triangles is shown below, starting with an area of 64.

\[
A_0 = 64 \quad A_1 = 48 \quad A_2 = ? \quad A_3 = ?
\]

(a) If we remove \( \frac{1}{4} \) of the area, what fraction of the area remains?

(b) Multiply 64 by the fraction you found in (a). What value do you get?

(c) Find the areas of the third and fourth pictures above by multiplying by the fraction you found in (a).

(d) Find a formula for the area, \( A \), that remains after \( n \) removals of area.

(e) How much area remains after 10 removals?

(f) How much area remains after 20 removals?

(g) Will the area ever reach zero? Explain your thinking.

(h) If the Sierpinski triangle to the right had an original area of 15 square centimeters before any area was removed, what is the area of the figure shown to the right to the nearest tenth of a square centimeter? Show the calculation that leads to your answer.
So far we have concentrated on linear functions which are characterized by having a constant rate of change. In the last lesson, we looked at exponential growth and decay. In this lesson we will more formally introduce the concept of an exponential function.

**Exercise #1:** Consider the exponential function $f(x) = 8(2)^x$. Answer the following.

(a) Evaluate each of the following and indicate what point must lie on the graph of $f(x)$ based on each:

(i) $f(2) =$

(ii) $f(0) =$

(iii) $f(-1) =$

(b) Calculate the average rate of change of $f$ over the interval $-1 \leq x \leq 0$.

(c) Calculate the average rate of change over the interval $0 \leq x \leq 2$.

(d) What does comparing answers from (b) and (c) tell you about this function? Explain.

(e) Using your calculator, draw a sketch of this function on the axes below using the window indicated.

Exponential functions are all about multiplication. The basic form of an exponential function is given below.

A general exponential function has the form: $y = a(b)^x$, where $a$ is the $y$-intercept and $b$ is the base or multiplying factor. Sometimes $b$ is known as the growth factor.
Let’s work some more with exponential functions to develop a better sense for them.

**Exercise #2:** Consider the function \( g(x) = 54\left(\frac{1}{3}\right)^x \).

(a) Evaluate \( g(0) \). What point does this indicate on the graph of \( g \)?

(b) Without the use of your calculator, determine the values of \( g(1) \) and \( g(2) \).

(c) Using your graphing calculator, sketch a graph of this function using the **WINDOW** \(-2 \leq x \leq 4\) and \(-10 \leq y \leq 100\). Mark the \( y \)-intercept.

(d) Why is this exponential function always **decreasing** while the one in Exercise #1 is always increasing?

---

**Exercise #3:** For each of the following exponential functions, give its \( y \)-intercept and tell whether it is increasing or decreasing.

(a) \( y = 8\left(\frac{2}{3}\right)^x \)  

(b) \( f(x) = 125\left(1.5\right)^x \)  

(c) \( P(t) = 56\left(\frac{3}{2}\right)^t \)

The equations of exponential functions are relatively easy to determine, if you understand this lesson so far. See what you can do in the next exercise.

**Exercise #4:** Find the equation of the exponential function, in \( y = a(b)^x \) form, for the function given in the table below. Show or explain your thinking.

<table>
<thead>
<tr>
<th>( x )</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>10</td>
<td>30</td>
<td>90</td>
<td>270</td>
<td>810</td>
</tr>
</tbody>
</table>
INTRODUCTION TO EXPONENTIAL FUNCTIONS
COMMON CORE ALGEBRA I HOMEWORK

FLUENCY

1. Consider the exponential function \( f(x) = 10(2)^x \).
   (a) Find the value of \( f(0) \). What point does this represent on the graph of \( y = f(x) \)?
   (b) Is this an increasing or decreasing exponential function? How can you tell based on its equation?
   (c) Is this function’s average rate of change over the interval \(-1 \leq x \leq 2\) greater or less than that of the linear function \( g(x) = 10x + 7 \)? Justify.
   (d) Using your calculator, sketch a graph of this function on the axes shown below. Use the window indicated. Mark the y-intercept.

```
<table>
<thead>
<tr>
<th>x</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>2</td>
<td>10</td>
<td>50</td>
<td>250</td>
<td>1250</td>
</tr>
</tbody>
</table>
```

2. Which of the following is a decreasing exponential function whose y-intercept is 20?
   (1) \( y = 20 \left( \frac{4}{3} \right)^x \)
   (2) \( y = 20 \left( \frac{1}{2} \right)^x \)
   (3) \( y = -2x + 20 \)
   (4) \( y = \left( \frac{1}{3} \right)^x + 20 \)

3. Which of the following functions would best describe the data in the table?
   (1) \( y = 10x + 2 \)
   (2) \( y = 8x + 2 \)
   (3) \( y = 5(2)^x \)
   (4) \( y = 2\left(5\right)^x \)
4. Graphing a basic exponential can be challenging because of how quickly they grow (or decay). In this exercise, we will graph one of the most basic.

\[ f(x) = 2^x \]

(a) Evaluate each of the following and state the coordinate point that occurs on the graph of \( f(x) \) based on the calculation.

\[ f(0) = \quad f(1) = \]
\[ f(2) = \quad f(3) = \]

(b) Evaluate each of the following. Remember your facts about negative exponents and give the point on the graph of \( f(x) \).

\[ f(-1) = \quad f(-2) = \quad f(-3) = \]

(c) Using the points you found in (a) and (b), graph this function for the domain interval \(-3 \leq x \leq 3\).

5. Classify each of the following exponential functions as either increasing or decreasing and give the value of their \( y \)-intercepts.

(a) \( y = 125(1.25)^x \)  
(b) \( y = 22 \left( \frac{3}{4} \right)^x \)  
(c) \( y = 256 \left( \frac{5}{2} \right)^x \)

**REASONING**

6. Which of the following could be the equation of the exponential function shown graphed below? Explain your choice.

(1) \( y = 15(1.25)^x \)  
(3) \( y = 50(1.04)^x \)

(2) \( y = 18(0.75)^x \)  
(4) \( y = 40(0.45)^x \)

Explanation:
One of the major topics that you studied in 7th Grade Common Core Math was the concept of a percent. Quite possibly, percents are the most applied and misunderstood concepts in mathematics. In today’s lesson, we will review the major ideas and problems dealing with percents. The main idea of percents is as follows.

A percent always compares two quantities as a proportional relationship out of 100.

**Exercise #1:** Jonathan is getting a raise from $12.50 per hour to $14.75 per hour. His supervisor, Makayla, got a raise from $22.00 per hour to $25.30 per hour.

(a) How much in terms of dollars per hour is Jonathan’s salary going up?  
(b) By what percent is Jonathan’s salary increasing?

(c) Who received the larger increase in salary in terms of dollars per hour?  
(d) Who received the larger percent increase in salary?

You can pretty much always solve percent problems by setting up proportional equations that involve 100.

**Exercise #2:** Gabe is buying a pair of jeans at a local store that are priced at $45. He knows that the county he lives in has an 8% sales tax added onto the list price. If Gabe has a gift card for $50, will it cover the cost of the jeans and the tax? Show work to justify your answer.

**Exercise #3:** The population of deer in a forest preserve is predicted to decline by 5% this year. If the current population is 560, what population is predicted for next year? What percent of the deer will remain?
Although it is convenient to solve percent problems using ratios, it is critical that you learn a different method. Some of you may have seen this before, if you’ve had teachers show it to you. If not, please make sure to understand what follows.

**Exercise #4:** Consider the following problem. Let’s say that I want to leave a 15% tip on a meal that cost $35.

(a) Find the tip by setting up a ratio involving 100. (b) Find the tip by doing a single multiplication problem. Why is this the same as (a)?

Know this important method for finding percents of totals:

**Finding Percents by Multiplying**

To find $p\%$ of a total, $T$, simply find the product: $\frac{P}{100} \cdot T$. Often $\frac{P}{100}$ is expressed as a decimal.

This “quick” way of finding percents of totals is a skill that you must become fluent with. Let’s get some practice with it in the next exercise.

**Exercise #5:** Find each of the following. Write down the product that you use to find your answer.

(a) 20% of 85  (b) 12% of 200  (c) 6% of 550

(d) 4.5% of 120  (e) 36% of 96  (f) $2\frac{1}{4}\%$ of 350

**Exercise #6:** Which of the following calculations would find 8.5% of 250?

(1) $(85)(250)$  (3) $(0.85)(250)$

(2) $(8.5)(250)$  (4) $(0.085)(250)$
PERCENT REVIEW
COMMON CORE ALGEBRA I HOMEWORK

FLUENCY

1. Evaluate the following percent problems by setting up and solving a ratio like we did in Exercises #1 through #3 in the lesson.
   (a) Find 7% of 280  
   (b) Find 12% of 300  
   (b) Find 2% of $1250

2. Find each of the following by a single multiplication problem (like what we did in Exercise #5 from the lesson). Write down the product that you use in your calculation.
   (a) Find 6% of 350  
   (b) Find 25% 80  
   (c) Find 15% of $35.00

3. Find each of the following by a single multiplication problem (like what we did in Exercise #5 from the lesson). These are trickier than #2. If needed, take the percent and divide it by 100 on your calculator to determine what to multiply by. Write down the product that you use in your calculation. Do not round your final answers.
   (a) 3.2% of 360  
   (b) 2.7% of 90  
   (c) 12.8% of 240
   (d) 0.8% of 450  
   (e) 0.5% of 500  
   (f) 0.25% of 320

4. If $x$ represents 2.8% of 270, then which of the following equation would not result in the correct value for $x$?
   (1) $\frac{x}{270} = \frac{2.8}{100}$
   (2) $x = \frac{2.8}{100} \cdot 270$
   (3) $x = (0.028)(270)$
   (4) $x = (0.28)(270)$
APPLICATIONS

5. Prestel currently makes $8.50 per hour. His boss has promised him a 15% raise in his hourly earnings.

(a) Calculate 15% of $8.50. Why can’t Prestel get exactly a 15% raise? What would his boss actually give him?

(b) After the raise, what is Prestel’s new salary? Show the calculation that leads to your answer.

6. Imani’s rent increased from $560 per month to $600 per month. Her friend, Ariana, had her rent increase from $825 to $875. Who had the larger percent increase in their rent? Remember to set up your ratios using the original rent. See Exercise #1 from the lesson if you want to see a similar problem.

7. The United States population is roughly 314 million people (314,000,000). The workforce participation rate, defined as the percent of the population working or looking for work, is 62.8%. The unemployment rate is the percent of the workforce that is looking for work, but cannot find it.

(a) How many people are working or looking for work in the United States?

(b) If the unemployment rate is currently 6.8%, then how many people are unemployed to the nearest hundred thousand.

REASONING

8. Niko had his savings increase by 5% this year. He started with $350 in his account and calculated how much he had at the end of the year by using the following sets of calculations:

\[ 350 \times 0.05 = 17.50 \quad 350 + 17.50 = 367.50 \]

Find a single number that Niko could have multiplied his starting amount of $350 by to get $367.50 by solving the equation below for \( r \). Why does this number make sense?

\[ 350r = 367.50 \]
In the last lesson we learned a quick way to find a percent of a total. In this lesson, we would like to develop and understand methods for increasing and decreasing a number by a certain percent. We will start with the increasing case.

**Exercise #1:** Agronomists are studying how quickly the population of an invasive species of beetle will increase in a controlled farm setting. They calculate that the population is increasing at a steady rate of 6% per week. At the beginning of the week, the population was 350 beetles.

(a) Find the population of beetles a week later by first finding 6% of 350 and adding it to the original population.

(b) Find the population of beetles a week later by a single multiplication. Why does this work?

(c) What will be the beetle population after two weeks?

The ability to increase a total by a certain percent using this method is important. Get some fluency with it in the next problem.

**Exercise #2:** Find the result of each of the following. Many of your answers will involve decimals. Do not round.

(a) Increasing 440 by 12%

(b) Increasing 68 by 8%

(c) Increasing 120 by 3.5%

**Exercise #3:** Adriana has a saving account that promises to increase her balance by 2.5% per year. If she deposits $720 in it at the beginning of the year, which of the following would be her balance at the end of the year if she does not withdraw or deposit any additional money?

(1) $900

(2) $842

(3) $738

(4) $756
We would also like to work with decreasing by a certain percent. This will follow a similar, if not identical, pattern to the increasing case. Again, let’s understand what is going on with an introductory problem.

**Exercise #4:** A cup of coffee is cooling down such that its temperature is decreasing at a constant rate of 8% per minute. Let’s say the coffee starts at a temperature of 200 °F.

(a) Find its temperature after one minute by finding 8% of 200 and then subtracting.

(b) Find its temperature after one minute by finding a single product. How can you interpret this in terms of the 8%?

(c) What will the temperature of the coffee be after two minutes? Round to the nearest degree.

**Decreasing** by a certain percent is an important skill to be fluent with as well. It often is harder for students because they need to think about what percent remains.

**Exercise #5:** The enrollment of students at a school is decreasing at a constant rate of 5% per year.

(a) What percent remains after one year? 

(b) If the population this year is 2300, what will its population be next year? Do in a single calculation.

**Exercise #6:** Find the result of each of the following. Many of your answers will involve decimals. Do not round. Write the calculations you use to find yours answers.

(a) Decrease 620 by 10% 

(b) Decrease $22.50 by 8% 

(c) Decrease 122 by 3.5%

**Exercise #7:** The cost of gasoline has decreased recently by 4.5%. If it started at $3.80 per gallon, which of the following is its price after the decrease?

(1) $3.63/gal  

(3) $3.22/gal  

(2) $3.72/gal  

(4) $3.97/gal
PERCENT INCREASE AND DECREASE
COMMON CORE ALGEBRA I HOMEWORK

FLUENCY

1. Perform each of the following calculations using a single multiplication. Show the product that you use to find your final answer. Do not round your final answers.
   (a) Increase 350 by 5%
   (b) Increase 120 by 10%
   (c) Increase 34 by 2%
   (d) Increase $450 by 3.5%
   (e) Increase $1,300 by $\frac{1}{26}$%
   (f) Increase 2,698 by $\frac{3}{42}$%

2. Perform each of the following calculations using a single multiplication. Show the product that you use to find your final answer. Do not round your final answers.
   (a) Decrease 160 by 10%
   (b) Decrease 450 by 6%
   (c) Decrease 122,000 by 12%
   (d) Decrease $1,820 by 3%
   (e) Decrease $12,500 by 15%
   (f) Decrease $4.50 by 8%

APPLICATIONS

3. A population of bacteria is growing at a rate of 20% per hour. If the population starts at 320, what is it an hour later?
   (1) 360
   (2) 356
   (3) 372
   (4) 384

4. The price of oil, in dollars per barrel, declined last week by 3.5%. If it started the week at $102.00 per barrel, at what per barrel price did it end the week?
   (1) $98.43
   (2) $98.50
   (3) $99.12
   (4) $100.56
5. A savings account grows by 3% per year. Sofia places $500 in the account at the beginning of the year.
   (a) How much does Sofia have at the end of the year once her $500 has been increased by 3%? 
   (b) How much does Sofia have at the end of the second year based on increasing your answer from part (a) by 3%?
   
   (c) Did the amount of money in Sofia’s account grow faster the first year or the second year? Explain how you arrived at your answer. Use proper units in your explanation.

6. An environmental firm is removing pollution from a river bed. They find that the pollution is decreasing at a rate of 5% per month, as measured by parts per million. The pollution starts off at a level of 360 parts per million (ppm).
   (a) What is the pollution level one month after they begin clean-up? Use appropriate units.
   (b) What is the level two months after? Use appropriate units.

**REASONING**

Percents build on one another in strange ways. It would seem that if you increased a number by 5% and then increased its result by 5% more, the overall increase would be 10%.

7. Let’s do exactly this with the easiest number to handle in percents.
   (a) Increase 100 by 5% 
   (b) Increase your result form (a) by 5%.

   (c) What was the overall percent increase of the number 100? Why is it not 10%?
There are many examples of growth in the real world that occur at a constant percent rate. These phenomena give rise to exponential functions. These functions will be easy to build and understand if you felt comfortable with the last lesson on percent growth and decay.

**Exercise #1:** A population of fruit flies is growing at a constant rate of 6% per hour. The population starts, at $t = 0$, with 28 flies.

(a) Using what we learned in the last lesson, determine the population after each of the following amounts of time. Show the calculation you use as repeated multiplication.

\begin{align*}
  t &= 1 \text{ hr} \quad P = \\
  t &= 2 \text{ hr} \quad P = \\
  t &= 3 \text{ hr} \quad P =
\end{align*}

(b) Based on (a), find a formula that models the population, $P$, as a function of the time in hours, $t$.

(c) What is the value of $P(24)$?

(d) What does the calculation you made in (c) represent about the fly population? State the range of the population function over the domain interval $0 \leq t \leq 24$.

Exponential growth is fairly easy to model and fairly easy to interpret in the model. Consider the following example.

**Exercise #2:** If the savings in a bank account can be modeled by the function $S(t) = 250(1.045)^t$. Which of the following is true?

1. The initial amount deposited was $250 and the interest earned is 45%.
2. The initial amount deposited was $2.50 and the interest rate is 4.5%.
3. The initial amount deposited was $250 and the interest rate is 4.5%.
4. The initial amount deposited was $2.50 and the interest rate is 45%.
We should also be able to model exponentially **decreasing** phenomena based on what we learned in the last lesson about percent decrease. Remember to always model based on the percent that remains.

**Exercise #3:** As water drains out of a pool, the depth of the water decreases at a constant percent rate of 20% per hour. The depth of the water, when the draining begins, is 12 feet.

(a) As in #1, find the depth, $D$, of the water in the pool after each of the following times, $t$.

\[
\begin{align*}
\text{t = 1 hr} & \quad D = \\
\text{t = 2 hr} & \quad D = \\
\text{t = 3 hr} & \quad D =
\end{align*}
\]

(b) Based on (a), create an equation that gives the depth, $D$, of the water in the pool as a function of the time in hours it has been draining, $t$.

(c) Using your calculator, sketch a graph of your function over the interval $0 \leq t \leq 20$ and $0 \leq D \leq 15$. Mark the $y$-intercept with its value.

(d) It’s safe to cover the pool after it reaches a depth of 1 foot or less. What is the minimum number of whole hours that we should wait to cover the pool? Explain how you found your answer.

**Exercise #4:** Which of the following equations would model an exponential quantity that begins at a level of 16 and decreases at a constant rate of 8% per hour?

(1) $Q = 16(0.92)^t$  
(2) $Q = 16 + 0.92^t$  
(3) $Q = 16(1.08)^t$  
(4) $Q = 16(-7)^t$

**Exercise #5:** If $350 is placed in a savings account that earns 3.5% interest applied once a year, then how much would the savings account be worth after 10 years?

(1) $522.88$  
(2) $426.34$  
(3) $472.50$  
(4) $493.71$
EXPONENTIAL MODELS BASED ON PERCENT GROWTH
COMMON CORE ALGEBRA I HOMEWORK

APPLICATIONS

1. An oil spill is spreading such that its area is given by the exponential function \( A(t) = 250(1.15)^t \), where \( A \) is the area in square feet and \( t \) is the time that has elapsed in days.
   (a) How large was the oil spill initially, i.e. at \( t = 0 \)?
   (b) By what percent is the oil spill increasing each hour?
   (c) Sketch a graph of the area of the oil spill over the interval \( 0 \leq t \leq 30 \) and \( 0 \leq A \leq 5000 \) using your calculator. Label the \( y \)-intercept.
   (d) After how many days will the oil spill reach a size of 3,000 square feet? Round to the nearest tenth. Solve graphically using the INTERSECT COMMAND on your calculator.
   (e) What is the average rate of change of \( A(t) \) over the interval \( 0 \leq t \leq 10 \)? Include proper units and don’t round.

2. If a flock of ducks is growing by 6% per year and starts with a population of 68, how many ducks will be in the flock after 10 years?
   (1) 109          (3) 122
   (2) 198          (4) 408

3. A bank account earns interest at a rate of 3.5% per year (in other words it increases in value by that percent) and starts with a balance of $350. Which of the following equations would give the account’s worth, \( W \), as a function of the number of years, \( y \), it has been gaining interest?
   (1) \( W = 350(1.035)^y \)          (3) \( W = 1.035y + 350 \)
   (2) \( W = 350(0.35)^y \)          (4) \( W = 1.35y + 350 \)
4. The amount, \( A \), in grams of a radioactive material that is decaying can be modeled by \( A(d) = 450(0.88)^d \), where \( d \) is the number of days since it started its decay.

(a) By what percent is the material decaying per day?

(b) Give an interpretation of the fact that \( A(14) = 75 \).

(c) Use your calculator to sketch a graph of \( A(d) \) over the interval \( 0 \leq d \leq 21 \). You determine an appropriate \( y \)-window and label the \( y \)-intercept with its value.

(d) The material is safe to transport once it has less than 5 grams of radioactive mass left. Using tables on your calculator, determine the first day when it will be safe to transport this material. Show some entries from your table to support your answer.

5. Newton’s Law of Cooling can be used to predict the temperature of a cooling liquid in a room that is at a certain steady temperature. We are going to model the temperature of a cooling cup of coffee. The Fahrenheit temperature of a cup of coffee, \( T \), in a room that is at a 72°F is given as a function of the number of minutes, \( m \), it has been cooling by:

\[
T(m) = 114(0.86)^m + 72
\]

(a) Find \( T(0) \) and using proper units, give a physical interpretation of your answer.

(b) What does the coefficient of 114 represent in terms of the situation being modeled?

(c) By what percent does the difference between the temperature of the coffee and the temperature of the room decrease each minute?

(d) I like my coffee when it is a nice temperature of around 100°F. How long should I wait?
Linear and exponential functions share many characteristics. This is because they are based on two different, but similar, sets of principles.

**LINEAR VERSUS EXPONENTIAL**

**Linear functions** are based on repeatedly adding the same amount (the slope).

**Exponential functions** are based on repeatedly multiplying by the same amount (the base).

**Exercise #1:** The two tables below represent a linear function and an exponential function. Which is which? Explain how you arrive at your answer.

<table>
<thead>
<tr>
<th>Table 1</th>
<th>Table 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x$</td>
<td>$x$</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>$y$</td>
<td>$y$</td>
</tr>
<tr>
<td>5</td>
<td>8</td>
</tr>
<tr>
<td>10</td>
<td>11</td>
</tr>
<tr>
<td>20</td>
<td>14</td>
</tr>
<tr>
<td>40</td>
<td>17</td>
</tr>
<tr>
<td>80</td>
<td>20</td>
</tr>
</tbody>
</table>

**Exercise #2:** Find equations in standard form for each of the functions from Exercise #1.

(a) Table 1

(b) Table 2

It is interesting that linear and exponential functions are ones where two points on the curve will always determine the equation of the curve.

**Exercise #3:** Consider the two points $(0, 12)$ and $(1, 3)$. Create a linear equation that passes through these points in $y = mx + b$ form and an exponential equation in $y = a(b)^x$ form that also passes through them. Then, using your calculator, graph both using a **WINDOW** of $-2 \leq x \leq 2$ and $-5 \leq y \leq 15$.
Recall that linear functions have a constant **average rate of change (slope)**. That’s, of course, why they have a constant amount added for every constant change in \( x \). Let’s examine the average rate of change for an increasing exponential.

**Exercise #4:** The exponential function \( f(x) = 4(2)^x \) is shown partially in the table below. Find the average rate of change over the various intervals given. This should be relatively simple because \( \Delta x = 1 \).

<table>
<thead>
<tr>
<th>( x )</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>4</td>
<td>8</td>
<td>16</td>
<td>32</td>
<td>64</td>
</tr>
</tbody>
</table>

(a) \( 0 \leq x \leq 1 \)  
(b) \( 1 \leq x \leq 2 \)  
(c) \( 2 \leq x \leq 3 \)  
(d) \( 3 \leq x \leq 4 \)

(e) What is clearly happening to the average rate of change as \( x \) gets larger?

The fact that the **slope** of an increasing exponential is **always increasing** has an interesting consequence.

**Exercise #5:** Consider the linear function \( y = 20x + 5 \) and the exponential function \( y = 5(2)^x \). Both of these functions have a \( y \)-intercept of 5, so “start” in the same location.

(a) Using your calculator, sketch these two curves on the axes below for the indicated window. Label each with its equation.

(b) Again, using your calculator, sketch these two curves on the axes below for the indicated window. Label each with its equation.

(c) Although the line appears to rise more quickly than the exponential, at first, the exponential eventually catches up and surpasses the linear. Why will an increasing exponential function always catch up with an increasing linear function?
LINEAR VERSUS EXPONENTIAL
COMMON CORE ALGEBRA I HOMEWORK

FLUENCY

1. For each of the following problems a table of values is given where \( \Delta x = 1 \). For each, first determine if the table represents a linear function, of the form \( y = mx + b \), or an exponential function, of the form \( y = a(b)^x \). Then, write its equation.

   (a) \[
   \begin{array}{c|cccc}
   x & -1 & 0 & 1 & 2 \\
   \hline
   y & 4 & 7 & 10 & 13 \\
   \end{array}
   \]

   Type: ____________________________

   Equation: _________________________

   (b) \[
   \begin{array}{c|cccc}
   x & 0 & 1 & 2 & 3 & 4 \\
   \hline
   y & 2 & 6 & 18 & 54 & 162 \\
   \end{array}
   \]

   Type: ____________________________

   Equation: _________________________

   (c) \[
   \begin{array}{c|cccc}
   x & -2 & -1 & 0 & 1 & 2 \\
   \hline
   y & 32 & 16 & 8 & 4 & 2 \\
   \end{array}
   \]

   Type: ____________________________

   Equation: _________________________

   (d) \[
   \begin{array}{c|cccc}
   x & -2 & -1 & 0 & 1 & 2 \\
   \hline
   y & 32 & 16 & 0 & -16 & -32 \\
   \end{array}
   \]

   Type: ____________________________

   Equation: _________________________

   (e) \[
   \begin{array}{c|cccc}
   x & 0 & 1 & 2 & 3 & 4 \\
   \hline
   y & 16 & 20 & 25 & 31\frac{1}{2} & 39\frac{1}{16} \\
   \end{array}
   \]

   Type: ____________________________

   Equation: _________________________

   (f) \[
   \begin{array}{c|cccc}
   x & 0 & 1 & 2 & 3 & 4 \\
   \hline
   y & 180 & 160 & 140 & 120 & 100 \\
   \end{array}
   \]

   Type: ____________________________

   Equation: _________________________

2. The data shown in the table below represents either a linear or an exponential function. Which of the equations below best models this data set?

   (1) \( y = 5(2)^x \) \quad (2) \( y = 10(2)^x \)

   (3) \( y = 2x + 10 \) \quad (4) \( y = 10x + 5 \)

   \[
   \begin{array}{c|cccc}
   x & 1 & 2 & 3 & 4 \\
   \hline
   y & 10 & 20 & 40 & 80 \\
   \end{array}
   \]
APPLICATIONS

3. Wildlife biologists are tracking the population of albino deer in an upstate New York forest preserve. They record the population every year since 2005, which they consider to be \( t = 0 \). Their data is shown in the table below.

<table>
<thead>
<tr>
<th>Year</th>
<th>2005</th>
<th>2006</th>
<th>2007</th>
<th>2008</th>
<th>2009</th>
<th>2010</th>
</tr>
</thead>
<tbody>
<tr>
<td>( t )</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>Population</td>
<td>86</td>
<td>98</td>
<td>111</td>
<td>128</td>
<td>147</td>
<td>168</td>
</tr>
</tbody>
</table>

(a) Although neither a linear nor an exponential function would model this data perfectly, justify why an exponential function would be a much better fit. Specifically, explain both why a linear function would not be a good fit while an exponential would be reasonable.

(b) Determine an equation for an exponential that models this data set in the form \( P = a(b)^t \).

(c) Use your model to predict the population of deer in the year 2014.

REASONING

4. You can determine the equation of a line or the equation of an exponential given any two points that lie on these curves. In this exercise we will pick two special points. Consider the points \((0, 5)\) and \((1, 15)\).

(a) Write the equation of the line that passes between these two points in \( y = mx + b \) form.

(b) Write the equation of the exponential that passes between these two points in \( y = a(b)^x \) form.

(c) Using your calculator, sketch the two curves on the axes below. Label with their equations.
GEOMETRIC SEQUENCES
COMMON CORE ALGEBRA I

In Unit #4 we first encountered sequences, which just consisted of a specific list of numbers in a particular order. We extensively studied the idea of an arithmetic sequence, where each successive number in the list was generated by adding the same quantity to the previous number. Let’s do a warm up.

**Exercise #1:** An arithmetic sequence is defined recursively by the following formula:

\[ a_1 = 5 \text{ and } a_n = a_{n-1} + 3 \]

(a) Find the next three terms of the sequence.  
(b) Find the value of \(a_{20}\) without listing out all 20 terms.

Clearly, arithmetic sequences share many characteristics of linear functions. In fact, arithmetic sequences are examples of discrete linear functions. Exponential functions have their own discrete versions and those are called geometric sequences. They have a very simple recursive definition.

**GEOMETRIC SEQUENCES**

Given the first term, \(a_1\), then each successive term can be found by \(a_i = a_{i-1} \cdot r\), where \(r\) is some constant often known as the common ratio of the sequence.

**Exercise #2:** For each of the following geometric sequences identify the common ratio, \(r\), and give the next two terms.

(a) 2, 6, 18, _______, _______  
(b) 4, –20, 100, _______, _______  
(c) 16, 8, 4, _______, _______

As with arithmetic sequences, we should be able to predict any particular term in the geometric sequence by thinking about how many times we have multiplied by the common ratio, \(r\).

**Exercise #3:** Consider the geometric sequence given by the recursive rule:

\[ b(1) = 3 \text{ and } b(n) = b(n-1) \cdot 2 \]

(a) Find \(b(2)\), \(b(3)\), and \(b(4)\). Write each as an extended product to see a pattern, but also find the final result.

(b) Based on (a), determine the value of \(b(10)\) and \(b(20)\).
One thing you might have noticed in the last exercise is how quickly a geometric sequence grows. Does this sound familiar? Let’s take a look at a classic problem.

**Exercise #4:** You have just won a very strange lottery. The lottery promises to give you money each day for a 30 day month based on one of two options:

**Option 1:** You can receive $1000 on the first day, $2000 on the second day, $3000 on the third, in this arithmetic sequence.

**Option 2:** You can receive $0.01 (one penny) on the first day, $0.02 on the second, $0.04 on the third, $0.08 on the fourth, etcetera in this geometric sequence.

Of the two options, which would result in the larger payoff on the 30th day only? Show work that supports your answer.

Graphs of geometric sequences will look familiar. Because they are a type of discrete exponential function they will look very similar.

**Exercise #5:** For a geometric sequence defined by \( a_1 = 16 \) and \( a_n = a_{n-1} \cdot \frac{1}{2} \), list and plot the first 6 terms on the grid below.
GEOMETRIC SEQUENCES
COMMON CORE ALGEBRA I HOMEWORK

FLUENCY

1. For each of the following geometric sequences, fill in the missing two terms and identify the common ratio, \( r \). Remember, you can always find \( r \) by dividing two consecutive terms such as \( \frac{a_2}{a_1} \).

(a) 2, 10, 50, _______ , _______  
\( r = \) _______

(b) 4, –8, 16, _______ , _______  
\( r = \) _______

(c) 40, 20, 10, _______ , _______  
\( r = \) _______

(d) 81, 54, 36, _______ , _______  
\( r = \) _______

(e) 5, –5, 5, _______ , _______  
\( r = \) _______

(f) 8, 20, 50, _______ , _______  
\( r = \) _______

2. One of the following sequences is arithmetic and one is geometric. Explain which is which.

Sequence #1: 5, 15, 45, 135, 405  
Sequence #2: 5, 15, 25, 35, 45

3. In a geometric sequence the first term is 5 and the second term is 20, which of the following is the fifth term?

(1) 65  
(2) 1,280  
(3) 80  
(4) 5,120
4. A geometric sequence is defined recursively by $a(1) = 40$ and $a(n) = a(n-1) \cdot \frac{1}{2}$.

   (a) Write out the first four terms of this sequence.  
   (b) Is the 9th term of this sequence larger or smaller than $\frac{1}{10}$? Show the calculation that you use to determine your answer.

5. Which has the larger 15th term when comparing the arithmetic and geometric sequences below? Show evidence that supports your answer.

   **Arithmetic Sequence:** 150, 650, 1150, 1650, …  
   **Geometric Sequence:** 4, 12, 36, 108, …

**APPLICATIONS**

6. Maria plans to double the amount of time she spends walking per day each week. She starts, on week 1, walking 5 minutes per day. After 7 days, she then walks 10 minutes per day, etcetera.

   (a) How many minutes per day will Maria be walking on Week #6? Show the calculation that gives your answer.

   (b) Scale the y-axis appropriately and graph the first six terms of this sequence. List them all if you haven’t already.

   (c) According to this geometric progression, how many minutes per day would Maria be walking on Week #10? Why is this not a viable answer?
UNIT #7

POLYNOMIALS

Lesson #1 – Introduction to Polynomials
Lesson #2 – Multiplying Polynomials
Lesson #3 – Factoring Polynomials
Lesson #4 – Factoring Based on Conjugate Pairs
Lesson #5 – Factoring Trinomials
Lesson #6 – More Work Factoring Trinomials
The way we write numbers in our systems is interesting because with only 10 digits, i.e. 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9, we are able to write whole numbers as large as we would like. This is because what we really are doing is counting how many powers of 10 that we have.

**Exercise #1**: Write each of the following numbers as a sum of multiples of powers of 10. The first is done as an example.

(a) \[563 = 500 + 60 + 3 = 5 \cdot 10^2 + 6 \cdot 10 + 3\]

(b) \[274\]

(c) \[3,842\]

(d) \[5,081\]

(e) \[21,478\]

We can now use algebra to replace the base of 10 with a generic base of \(x\) (or whatever variable you like).

**Exercise #2**: Consider the number 63,735.

(a) As in #1, write this number as the sum of multiples of powers of 10.

(b) If \(x = 10\), write this number in terms of an equivalent expression involving \(x\).

The base of a polynomial certainly doesn’t have to be 10. But, all polynomials have a form similar to your answer in letter (b). Let’s define them a little more definitively.

### Polynomial Expressions

Any expression of the form: \[a x^n + b x^{n-1} + c x^{n-2} + \cdots + \text{constant},\] where the exponents, \(n, n-1, n-2, \text{etcetera}\) are all positive integers. Note that not all powers need to be presents because the coefficients, i.e. \(a, b, c, \text{etcetera}\) can be zero.

**Exercise #3**: Of the expressions shown below, circle all of them that represent polynomials. Discuss why the ones that aren’t polynomials fail the definition above.

- \[4x^2 + 8x + 1\]
- \[9x^2 + 2x + \frac{1}{x}\]
- \[2^x + 3^x + 4^x\]
- \[2x^2 + 5x^3 - x + 8\]
It is often important to place polynomials in their **standard form**. The standard form of a polynomial is simply achieved by writing it as an **equivalent expression** where the powers on the variables **always descend**.

**Exercise #4:** Write each of the following polynomials in standard form.

(a) \(3x^2 + 5x^3 + 7 - 8x\)  
(b) \(9x^4 + 2x - x^2 + 1\)  
(c) \(3 - 2x - 5x^2\)

Polynomials are simply abstract representations of numbers that we see every day and they behave like these numbers as well. Let’s look at adding polynomials together.

**Exercise #5:** Consider the numbers 523 and 271.

(a) Write each as the sum of multiples of powers of 10 as previously done.  
(b) Add these numbers by adding each individual power of 10.

(c) Use this idea to add: \(5x^2 + 2x + 3 + 2x^2 + 7x + 1\)  
(d) Find the sum of the polynomials \(-4x^2 + 9x - 3\) and \(7x^2 - 5x + 4\).

Finding sums of polynomials is fairly easy. Subtracting them, though, can lead to a lot of errors.

**Exercise #6:** Find each of the following differences. Be careful and rewrite as an equivalent addition problem if necessary.

(a) \(6x^2 + 5x + 3 - 2x^2 - 4x + 7\)  
(b) \((4x^2 - 2x + 7) - (-2x^2 + x - 3)\)

**Exercise #7:** For each of the following, write an equivalent polynomial in simplest standard form.

(a) \(6x^2 + 2x - 3 - x^2 + 4x - 1\)  
(b) \(6x^2 + 2x - 3 - (x^2 + 4x - 1)\)
INTRODUCTION TO POLYNOMIALS
COMMON CORE ALGEBRA I HOMEWORK

FLUENCY

1. Write each of the following integers as multiples of powers of 10. The first is done as a reminder of this process.

(a) 563  
563 = 500 + 60 + 3 
= 5 \cdot 100 + 6 \cdot 10 + 3 
= 5 \cdot 10^2 + 6 \cdot 10 + 3 

(b) 278 

(c) 703 

(d) 5,378 

(f) 19,073 

2. Consider the number 5,364.

(a) Write this number as the sum of multiples of powers of 10 as in #1.

(b) If \( x = 10 \), write an expression in terms of \( x \) for the number 5,364.

3. Which of the following would be the value of the expression \( 5x^3 + 2x^2 + 8x + 4 \) when \( x = 10 \)?

(1) 6,432  
(2) 2,854  
(3) 5,284  
(4) 528

4. Which of the following would be the value of the expression \( 8x^3 + 2x + 3 \) when \( x = 10 \)?

(1) 823  
(2) 8,023  
(3) 8,203  
(4) 8,230

5. Which of the following is not a polynomial expression?

(1) \( x^4 \)  
(2) \( 3^x \)  
(3) \( 1 - 2x^3 \)  
(4) \( 6x + 1 \)
6. Write each of the following polynomial expressions in standard form.

(a) \(7x^2 + 4x^3 + 5 + 2x\)  
(b) \(4 - x - 5x^2\)  
(c) \(x^3 + x - 7x^2 + 2\)

(d) \(2x + 1 - 3x^3 + 5x^2\)  
(e) \(4x^3 - 2x^2 + 6 - 8x\)  
(f) \(y^5 + y^{10} - y^2 + y^7\)

7. Find each of the following sums and differences. Write your answer in simplest standard form.

(a) \(6x^2 - 2x + 8 + 3x^2 + 7x - 2\)  
(b) \(x^3 + 4x^2 - 8x + 3 + x^3 - x + 1\)

(c) \((5x^2 + 3x - 1) - (3x^2 - 6x + 4)\)  
(d) \((2x^3 - 5x^2 + 8x - 1) - (-4x^3 + 8x^2 - 3x - 9)\)

(e) \(4x^2 + 6x - 3 - 3x^2 + 2x + 4\)  
(f) \((4x^2 + 6x - 3) - (3x^2 + 2x + 4)\)

**APPLICATIONS**

8. A box has a width that is 2 inches greater than its height and a length that is 6 inches greater than its height. It’s volume is given by the polynomial expression \(x^3 + 8x^2 + 12x\), where \(x\) is the box’s height. What is the box’s volume, in cubic inches, if its height is 10 inches?

(1) 1,812  
(2) 1,920  
(3) 182  
(4) 2,180

**REASONING**

9. Polynomial expressions act a lot like integers because the structure of polynomials is based on the structure of integers. Based on the statement below about integers, make a statement about polynomials.

**Statement About Integers:** An integer added to an integer gives an integer.

**Statement About Polynomials:**
Polynomials, as we saw in the last lesson, behave a lot like integers (whole numbers including the negatives). We saw that just like integers, adding one polynomial to another polynomial results in a third polynomial. The same will occur with multiplying them. First, a review problem.

**Exercise #1:** Monomials are the simplest of polynomials. They consists of one term (terms are separated by addition and subtraction). Find the following products of monomials.

(a) $5x^3 \cdot 2x^2$  
(b) $-3x \cdot -8x$  
(c) $\frac{1}{2}x^2y^5 \cdot \frac{3}{4}x^9y$

We have also used the **Distributive Property** in previous lessons to multiply polynomials that are more complicated.

**Exercise #2:** Find each of the following products in simplest form by using the distributive property once or twice.

(a) $2x(3x-1)$  
(b) $x^2(4x^2 + 3)$  
(c) $-2x^2y^3(2xy - 5x)$

(d) $(x+2)(x-6)$  
(e) $(2x+7)(x+3)$  
(f) $(3x-2)(5x-1)$

Never forget that as we do these manipulations we are using **properties of equality** to produce equivalent expressions.

**Exercise #3:** Consider the product of the two binomial polynomials $(x-1)(x-3)$.

(a) Find this product and express it as a **trinomial polynomial** written in standard form. Fill in the result in the first row (third column) of table (b).

(b) Fill out the table below using **TABLES** on your calculator to show they are equivalent.
We can evaluate more complicated products, just as we have done in the past with normal numbers. The key will always be the careful use of the **distributive property**.

**Exercise #4:** Find each of the following more challenging products.

(a) \((2x + 5)^2\)  
(b) \((x + 2)(x^2 + 4x + 3)\)

(c) \((x - 4)(x + 3)(x - 5)\)  
(d) \((3x + 2)^3\)

**Exercise #5:** Consider the product \((3x + 2)(2x + 1)\).

(a) Write this product as an equivalent trinomial expression in standard form.  
(b) How can you use your answer from (a) to evaluate the product \((32)(21)\)? Find the product and check using your calculator.
MULTIPLYING POLYNOMIALS
COMMON CORE ALGEBRA I HOMEWORK

FLUENCY

1. Write the following products as polynomials in either \(x\) or \(t\). The first is done as an example for you.

   (a) \(5x(2x-4)\)
   
   \[= (5x)(2x) - (5x)(4)\]
   
   \[= (5\cdot2)(x\cdot x) - (5\cdot4)(x)\]
   
   \[= 10x^2 - 20x\]

   (b) \(3t(t+7)\)
   
   (c) \(-4x(5x+1)\)

   (d) \(4(t^2-5t+2)\)
   
   (e) \(x(x^2-2x-3)\)
   
   (f) \(-5t(2t^2+3t-7)\)

2. Perhaps the most important type of polynomial multiplication is that of two binomials. Make sure you are fluent with this skill. Write each of the following products as an equivalent polynomial written in standard form. The first problem is done as an example using repeated distribution.

   (a) \((x+5)(x-3)\)
   
   \[= (x+5)(x) + (x+5)(-3)\]
   
   \[= (x)(x) + (5)(x) + (x)(-3) + (-5)(3)\]
   
   \[= x^2 + 5x - 3x - 15\]
   
   \[= x^2 + 2x - 15\]

   (b) \((x-10)(x-4)\)
   
   (c) \((x+3)(x+12)\)

   (d) \((2x+3)(5x+8)\)
   
   (e) \((4x-1)(x+2)\)
   
   (f) \((6x-5)(4x-3)\)

3. Never forget that squaring a binomial also a process of repeated distribution. Write each of the following perfect squares as trinomials in standard form.

   (a) \((x+3)^2\)
   
   (b) \((x-10)^2\)
   
   (c) \((2t+3)^2\)
4. An interesting thing happens when you multiply two *conjugate binomials*. Conjugates have the property of having the same *terms* but differ by the operation between the two terms (in one case addition and in one case subtraction). Multiply each of the following *conjugate pairs* and state your answers in *standard form*. The first is done as an example

(a) \((x + 3)(x - 3)\)  
(b) \((x - 5)(x + 5)\)  
(c) \((10 + x)(10 - x)\)  

\[ 
\begin{align*}
(a) & \quad (x + 3)(x - 3) = x(x - 3) + 3(x - 3) = x^2 - 3x + 3x - 9 = x^2 - 9 \\
(b) & \quad (x - 5)(x + 5) = x^2 - 5x + 5x - 25 = x^2 - 25 \\
(c) & \quad (10 + x)(10 - x) = 100 - x^2
\end{align*}
\]

5. Write each of the following products in standard polynomial form.

(a) \((x + 3)(x - 2)(x - 8)\)  
(b) \((x + 2)(x - 2)(x + 3)(x - 3)\) (Hint: try to use #4)

**Reasoning**

6. Notice again how similar polynomials are to integers, i.e. the set \(\{\ldots -3, -2, -1, 0, 1, 2, 3 \ldots\}\). Write a statement below for polynomials based on the statement about integers.

**Statement About Integers:** An integer times an integer produces an integer.

**Statement About Polynomials:** _____________________________________________________________  
___________________________________________________________

7. Consider the product \((3x + 1)^2\).

(a) Write this product in standard trinomial form.  
(b) Use your answer in part (a) to determine the value of \(31^2\) without your calculator.
FACTORING POLYNOMIALS
COMMON CORE ALGEBRA I

Factoring expressions is one of the gateway skills that is necessary for much of what we do in algebra for the rest of the course. The word factor has two meanings and both are important.

THE TWO MEANINGS OF FACTOR

1. Factor (verb): To rewrite an algebraic expression as an equivalent product.
2. Factor (noun): An algebraic expression that is one part of a larger factored expression.

Exercise #1: Consider the expression $6x^2 + 15x$.

(a) Write the individual terms $6x^2$ and $15x$ as completely factored expressions. Determine their greatest common factor.

$$6x^2 = \quad 15x =$$

(b) Using the Distributive Property, rewrite $6x^2 + 15x$ as a product involving the gcf from (a).

(c) Evaluate both $6x^2 + 15x$ and the factored expression you wrote in (b) for $x = 2$. What do you find? What does this support about the two expressions?

It is important that you are fluent reversing the distributive property in order to factor out a common factor (most often the greatest common factor). Let’s get some practice in the next exercise just identifying the greatest common factors.

Exercise #2: For each of the following sets of monomials, identify the greatest common factor of each. Write each term as an extended product (if necessary).

(a) $12x^3$ and $18x$  
(b) $5x^4$ and $25x^2$  
(c) $21x^2y^5$ and $14xy^7$

(d) $24x^3$, $16x^2$, and $8x$  
(e) $20x^3$, $-12x^2$, and $28x$  
(f) $18x^2y^2$, $45x^2y$, and $90xy^2$
Once you can identify the greatest common factor of a set of monomials, you can then easily use it and the distributive property to write equivalent factored expressions.

**Exercise #3:** Write each polynomial below as a factored expression involving the greatest common factor of the polynomial.

(a) $6x^2 + 10x$  
(b) $3x - 24$  
(c) $10x^2 - 15x$

(d) $4x^2 + 8x + 24$  
(e) $6x^3 - 8x^2 + 2x$  
(f) $10x^3 - 35x^2$

(g) $10x^2 - 40x - 50$  
(h) $8x^4 - 2x^2$  
(i) $8x^3 + 24x^2 - 32x$

Being able to **fluently** factor out a gcf is an essential skill. Sometimes greatest common factors are more complicated than simple monomials. We have done this type of factoring back in Unit #1.

**Exercise #4:** Rewrite each of the following expressions as the product of two binomials by factoring out a common binomial factor.

(a) $(x + 5)(x - 1) + (x + 5)(2x - 3)$  
(b) $(2x - 1)(2x + 7) - (2x - 1)(x - 3)$
FACTORING POLYNOMIALS
COMMON CORE ALGEBRA I HOMEWORK

FLUENCY

1. Identify the greatest common factor for each of the following sets of monomials.
   (a) $6x^2$ and $24x^3$
   (b) $15x$ and $10x^2$
   (c) $2x^4$ and $10x^2$
   (d) $2x^3$, $6x^2$, and $12x$
   (e) $16t^2$, $48t$, and $80$
   (f) $8t^5$, $12t^3$, and $16t$

2. Which of the following is the greatest common factor of the terms $36x^2y^4$ and $24xy^7$?
   (1) $12xy^4$
   (2) $24x^2y^7$
   (3) $6x^2y^3$
   (4) $3xy$

3. Write each of the following as equivalent products of the polynomial’s greatest common factor with another polynomial (of the same number of terms). The first is done as an example.
   (a) $8x - 28$
   (d) $18 - 12x$
   (g) $10x^2 + 35x - 20$
   (j) $30x^3 - 75x^2$
   (b) $50x + 30$
   (e) $6x^3 + 12x^2 - 3x$
   (h) $21x^3 - 14x$
   (k) $-16t^2 + 96t$
   (c) $24x^2 + 32x$
   (f) $x^2 - x$
   (i) $36x - 8x^2$
   (l) $4t^3 - 32t^2 + 12t$

4. Which of the following is not a correct factorization of the binomial $10x^2 + 40x$?
   (1) $10x(x + 4)$
   (2) $10(x^2 + 4x)$
   (3) $5x(2x + 4)$
   (4) $5x(x + 8)$
5. Rewrite each of the following expressions as the product of two binomials by factoring out a common binomial factor. Watch out for the subtraction problems (b) and (d).

(a) \((x + 5)(x+1) + (x + 5)(x+8)\)  
(b) \((2x-1)(3x+5)-(2x-1)(x+4)\)

(c) \((x-7)(x-9) + (x-7)(4x+5)\)  
(d) \((x+1)(5x-7)-(x+1)(x-3)\)

APPLICATIONS

6. The area of a rectangle is represented by the polynomial \(16x^2 + 56x\). The width of the rectangle is given by the binomial \(2x + 7\).

(a) Give a monomial expression in terms of \(x\) for the length of the rectangle. Show how you arrived at your answer.  
(b) If the length of the rectangle is 80, what is the width of the rectangle? Explain your thinking.

REASONING

7. These crazy polynomials keep acting like integers. We can factor integers to determine their factors. We can also do the same for polynomials.

(a) List all of the positive factors of the integer 12 by writing all possible positive integer products (such as \(12 = 3 \cdot 4\)).  
(b) List all of the factors of \(2x^2 - 6x\) by also writing all possible products, such as \(2(x^2 - 3x)\).

8. Which of the following is not a factor of \(4x^2 + 12x\)?

(1) \(x + 3\)  
(2) \(x\)  
(3) \(3x\)  
(4) \(4\)
COMMON CORE ALGEBRA I, UNIT #7 – POLYNOMIALS – LESSON #4

FACTORYING BASED ON CONJUGATES
COMMON CORE ALGEBRA I

There are a number of different types of factoring techniques. But, each one of them boils down to reversing a product. We begin the lesson today by looking at products of conjugate binomials, or binomials of the form \(a + b\) and \(a - b\).

**Exercise #1:** Find each of the following products of conjugate pairs. See if you can work out a pattern.

(a) \((x+5)(x-5)\) 
(b) \((x-2)(x+2)\) 
(c) \((4x+1)(4x-1)\)

(d) \((x+y)(x-y)\) 
(e) \((2x+3)(2x-3)\) 
(f) \((5x+2y)(5x-2y)\)

What we should see is that if we multiply conjugates, opposites always cancel and instead of getting our expected trinomial, we still get a binomial. Specifically.

**MULTIPLYING CONJUGATE PAIRS**

\[(a+b)(a-b) = a^2 - b^2\]

**Exercise #2:** Use the pattern from Exercise #1 to quickly rewrite the following products.

(a) \((x+6)(x-6)\) 
(b) \((5x+2)(5x-2)\) 
(c) \((2x+7y)(2x-7y)\)

(d) \((4+x)(4-x)\) 
(e) \((6+5y)(6-5y)\) 
(f) \((10x-4y)(10x+4y)\)
We now should be able to reverse this multiplication in order to rewrite expressions that are the difference of perfect squares into products.

**Exercise #3:** Write each of the following first in the form $a^2 - b^2$ and then as equivalent products of conjugate pairs.

(a) $x^2 - 81$

(b) $9x^2 - 4$

(c) $25 - y^2$

(d) $4x^2 - 81y^2$

(e) $121x^2 - 1$

(f) $1 - 4x^2$

Never forget that when we factor, we are always rewriting an expression in a form that might look different, but it is ultimately still equivalent to the original.

**Exercise #4:** Let’s take a look at the binomial $x^2 - 9$.

(a) Amelia believes that $x^2 - 9$ can be factored as $(x + 1)(x - 9)$ while her friend Isabel believes that it is factored as $(x - 3)(x + 3)$. Fill out the table below to develop evidence as to who is correct. Use technology on your calculator to help.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$x^2 - 9$</th>
<th>$(x + 1)(x - 9)$</th>
<th>$(x - 3)(x + 3)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(b) By multiplying out their respective factors, show which of the two friends has the correct factorization. Use the Distributive Property Twice.

Amelia: $(x + 1)(x - 9)$  
Isabel: $(x - 3)(x + 3)$
FACTORING BASED ON CONJUGATE PAIRS
COMMON CORE ALGEBRA I HOMEWORK

FLUENCY

1. Use the fact that the product of conjugates follows the following pattern, \((a + b)(a - b) = a^2 - b^2\), to quickly find the following products in standard form.

(a) \((x - 5)(x + 5)\)  
(b) \((x + 7)(x - 7)\)  
(c) \((2 - x)(2 + x)\)

(d) \((3x + 2)(3x - 2)\)  
(e) \((4x + 1)(4x - 1)\)  
(f) \((2x + 1)(2x - 1)\)

(g) \((5 - 4x)(5 + 4x)\)  
(h) \((x^2 - 2)(x^2 + 2)\)  
(i) \((x^3 + 4)(x^3 - 4)\)

2. Write each of the following binomials as an equivalent product of conjugates.

(a) \(x^2 - 16\)  
(b) \(x^2 - 100\)  
(c) \(x^2 - 1\)

(d) \(x^2 - 25\)  
(e) \(4 - x^2\)  
(f) \(9 - x^2\)

(g) \(4x^2 - 1\)  
(h) \(16x^2 - 49\)  
(i) \(1 - 25x^2\)

(j) \(x^2 - 9y^2\)  
(k) \(81 - 4t^2\)  
(l) \(x^4 - 36\)
APPLICATIONS

3. A square is changed into a new rectangle by increasing its width by 2 inches and decreasing its length by 2 inches. Make sure to draw pictures to help you solve these problems!

(a) If the original square had a side length of 8 inches, find its area and the area of the new rectangle. How many square inches larger is the square’s area?

(b) If the original square had a side length of 20 inches, find its area and the area of the new rectangle. How many square inches larger is the square’s area?

(c) If the square had a side length of \( x \) inches, show that its area will always be four square inches more than the area of the new rectangle.

REASONING

4. Consider the numerical expression \( 51^2 - 49^2 \).

(a) Use your calculator to find the numerical value of this expression.

(b) Can you used facts about conjugate pairs to show why this difference should work out to be the answer from (a)?

5. Consider the following expression \( (x + 2)(x - 2) - (x + 4)(x - 4) \).

(a) Using your calculator, determine the value of this expression for various values of \( x \).

(b) Algebraically show that this product has a constant value (seen in (a)) regardless of the value of \( x \).
So far we have two factoring techniques: (1) Factoring out a g.c.f. and (2) Factoring based on conjugate pairs (factoring the difference of perfect squares). Today we will tackle the most difficult of the factoring techniques, and that is of factoring trinomials. First, let’s make sure we can multiply binomials.

**Exercise #1:** Write each of the following products in equivalent trinomial form.

(a) \((x + 5)(x + 3)\) \hspace{1cm} (b) \((x - 2)(x + 7)\) \hspace{1cm} (c) \((x - 5)(x - 10)\)

Try a few that are a bit more difficult. It is critical that you are fluent with this type of multiplication before we try to reverse the process.

**Exercise #2:** Write each of the following products in equivalent trinomial form.

(a) \((2x - 3)(5x + 1)\) \hspace{1cm} (b) \((6x + 7)(x + 2)\) \hspace{1cm} (c) \((4x - 1)(2x - 5)\)

Now, we need to reverse this process to take a trinomial and write it as the product of the binomials. There is truly only one fail proof method for this type of factoring and that is GUESS AND CHECK. This method often frustrates students, but look at it as a puzzle and make “smart” guesses and quick checks.

**Exercise #3:** Consider the trinomial \(2x^2 + 13x + 20\). We want to write it as the product of two binomials, in other words, reverse what we did in Exercises #1 and #2.

(a) Fill in the missing blanks with the only pair that makes sense (that is a smart guess).

\[\text{ } (\underline{\hspace{1cm}} + \underline{\hspace{1cm}})(\underline{\hspace{1cm}} + \underline{\hspace{1cm}}).\]

(b) Why does it make sense that both of the binomials will be addition (as shown in (a))? 

(c) List pairs of factors that can produce a 20.

(d) Try various pairs from (c), checking only that the linear terms will sum to 13x until you find the correct factorization.
There is absolutely no substitution for rote practice with factoring trinomials. To be able to do this critical skill, you must be smart with your guesses and patient with your checks. You will find the correct answer, but you must be willing to guess incorrectly multiple times.

**Exercise #4:** Write each of the following trinomials in equivalent factored form. Show your checks and don’t worry if your first guess isn’t correct. You will get better at these! Note for yourself which ones seemed easier and which were harder.

(a) $3x^2 + 11x - 4$  
(b) $x^2 - 7x + 10$

(c) $x^2 + 10x + 21$  
(d) $10x^2 + 9x + 2$

(e) $8x^2 - 18x + 9$  
(f) $2x^2 + 5x - 33$

(g) $x^2 - 8x + 12$  
(h) $7x^2 - 26x - 8$
FACTORYING TRINOMIALS
COMMON CORE ALGEBRA I HOMEWORK

FLUENCY

1. Which of the following products is equivalent to the trinomial \(x^2 - 5x - 24\) ?
   (1) \((x-12)(x+2)\)  (3) \((x-8)(x+3)\)
   (2) \((x+12)(x-2)\)  (4) \((x+8)(x-3)\)

2. Written in factored form, the trinomial \(2x^2 + 15x + 28\) can be expressed equivalently as
   (1) \((2x+7)(x+4)\)  (3) \((2x+2)(x+14)\)
   (2) \((2x+4)(x+7)\)  (4) \((2x+14)(x+2)\)

3. (Easier) Write each of the following trinomials in equivalent factored form. Keep at it!!! Use extra scrap paper for extra guesses.
   (a) \(x^2 + 10x + 16\)  (b) \(x^2 + 12x + 35\)  (c) \(x^2 + 12x + 36\)
   (d) \(x^2 - 5x + 6\)  (e) \(x^2 - 11x + 28\)  (f) \(x^2 - 12x + 20\)
   (g) \(x^2 - 3x - 18\)  (h) \(x^2 + 3x - 40\)  (i) \(x^2 - 10x - 24\)
   (j) \(x^2 - 8x + 15\)  (k) \(x^2 + 30x + 200\)  (l) \(x^2 + 8x - 9\)
4. (Medium) Write each of the following trinomials in equivalent factored form. Keep at it!!! Use extra scrap paper for extra guesses.

(a) \(2x^2 + 13x + 21\) \hspace{1cm} (b) \(5x^2 - 21x + 4\) \hspace{1cm} (c) \(3x^2 + 16x - 12\)

(d) \(7x^2 + 11x - 6\) \hspace{1cm} (e) \(2x^2 - x - 10\) \hspace{1cm} (f) \(11x^2 - 10x - 1\)

5. (Hardest) Write each of the following trinomials in equivalent factored form. Keep at it!!! These might require quite a few tries. You will get better at them as you practice. Use extra scrap paper for extra guesses.

(a) \(4x^2 + 27x + 18\) \hspace{1cm} (b) \(6x^2 + 5x - 4\) \hspace{1cm} (c) \(12x^2 - 31x + 20\)

APPLICATIONS

6. A rectangle has dimensions as shown below in terms of an unknown variable, \(x\).

(a) Find a binomial expression for the width of the rectangle in terms of \(x\). Justify your answer based on the expressions for the rectangle’s length and area.

\[
\text{Length} = x + 4
\]

\[
\text{Area} = 2x^2 + 9x + 4
\]

\[
\text{Width} = ?
\]

(b) If the width of the rectangle is 21 inches, what is the length and the area? Use appropriate units and explain how you found your answer.
MORE WORK FACTORING TRINOMIALS
COMMON CORE ALGEBRA I

Factoring trinomials, which we first practiced in the last lesson, is a trying experience. All algebra students must learn how to do this procedure because of its immense number of practical applications. We will eventually see these applications, but for now, we need to get more practice factoring these trinomials. We begin by looking at a process known as complete factoring.

Exercise #1: Consider the trinomial $4x^2 + 20x + 24$.

(a) Write this trinomial as an equivalent expression involving the product of its term’s gcf and another trinomial.

(b) Factor this additional trinomial to express the original in completely factored form.

Whenever we factor, we should always look to see if a greatest common factor exists that can be “factored out” to begin the problem. This will always make any subsequent factoring easier.

Exercise #2: Rewrite each of the following trinomials in completely factored form.

(a) $10x^2 + 15x - 10$  
(b) $3x^3 - 21x^2 + 36x$

(c) $7x^2 + 21x - 70$  
(d) $6x^2 - 2x - 4$
Complete factoring can also involve factoring the **difference of perfect squares**. Try the next exercise to see how this works.

**Exercise #3:** Write each of the following binomials in completely factored form.

(a) \(2x^2 - 18\)  
(b) \(5x^3 - 20x\)

(c) \(12x^2 - 3\)  
(d) \(54x^2 - 24\)

If you understand factoring as breaking an expression into an equivalent product, then essentially you can always check to see if you have factored correctly. Complete factoring actually leads to a nice way to eliminate some guesses from trinomial guess and check methods.

**Exercise #4:** Consider the trinomial \(2x^2 + 11x + 12\).

(a) Do the three terms of this trinomial have a gcf other than 1?  
(b) Why would the guesses \((2x+2)(x+6)\), \((2x+4)(x+3)\), and \((2x+12)(x+1)\) not make sense given your answer to (a)?

(c) Fill in the statement:

   If a trinomial does not have a gcf, then
   
   \[\text{__________ of its __________ factors will have a gcf.}\]

   (d) Factor this trinomial by limiting your guesses.

**Exercise #5:** Use the Smart Guessing Tip from the last problem to factor \(4x^2 - 21x - 18\).
MORE WORK FACTORING TRINOMIALS
COMMON CORE ALGEBRA I HOMEWORK

FLUENCY
1. Rewrite each of the following trinomials in completely factored form.
   (a) \(2x^2 + 20x + 42\)
   (b) \(6x^2 + 33x + 15\)
   (c) \(5x^2 - 10x - 40\)
   (d) \(30x^2 + 20x - 10\)
   (e) \(x^3 + 7x^2 + 10x\)
   (f) \(4x^3 + 10x^2 - 24x\)
   (g) \(5x^2 - 45\)
   (h) \(2x^3 - 2x\)
   (i) \(36 - 4x^2\)
   (j) \(20x^2 - 125\)

2. Which of the following is \textit{not} a factor of \(4x^3 + 12x^2 - 72x\)? Show work that justifies your choice.
   (1) \((x + 9)\)
   (2) \(4x\)
   (3) \((x - 3)\)
   (4) \((x + 6)\)
3. Which of the following is the missing factor in the product $2(x-1)(\ ? \ )$ if it is equivalent to the trinomial $2x^2+10x-12$?

(1) $x+12$  
(2) $x+6$  
(3) $x+3$  
(4) $x-5$

4. Use the Smart Guessing Tip from Exercise #4 to help factor the following challenging trinomials. Note that they do **not** have a greatest common factor.

(a) $4x^2+19x+12$  
(b) $6x^2+7x-24$

**REASONING**

5. Consider the **cubic trinomial** $x^3+8x^2+7x$.

(a) Write this trinomial as an equivalent product in completely factored form.

(b) How can the original trinomial and your answer to (b) help you determine the value of $(10)(17)(11)$ without a calculator? What is the value?

6. Use the complete factorization of $2x^3+8x^2+8x$ to determine the value of the product $(20)(12)^2$. Explain your reasoning.
UNIT #8

QUADRATIC FUNCTIONS AND THEIR ALGEBRA

Lesson #1 – Introduction to Quadratic Functions
Lesson #2 – More Work with Parabolas
Lesson #3 – The Shifted Form of a Parabola
Lesson #4 – Completing the Square
Lesson #5 – Stretching Parabolas and More Completing the Square
Lesson #6 – The Zeroes of a Quadratic
Lesson #7 – More Zero Product Law Work
Lesson #8 – Quadratic Word Problems
INTRODUCTION TO QUADRATIC FUNCTIONS
COMMON CORE ALGEBRA I

We have now studied linear and exponential functions. These functions were relatively simple because they were either always increasing or always decreasing for their entire domains. We now will start to study other functions, most notably quadratic functions, which are a type of polynomial function. Their definition is shown below:

**QUADRATIC FUNCTIONS**

Any function that can be placed in the form: \( y = ax^2 + bx + c \), where \( a \neq 0 \), but \( b \) and \( c \) can be zero.

**Exercise #1:** Read the definition above for quadratic functions and answer the following questions.

(a) Why is it important for the leading coefficient to be nonzero?

(b) Circle the choices below that are quadratic functions.

\[
\begin{align*}
  y &= x^2 - 3 \\
  y &= x^3 + 2x^2 - 4 \\
  y &= x^2 + \sqrt{x} + 7 \\
  y &= 10 - x^2
\end{align*}
\]

(c) Given the quadratic function \( y = 10 - 3x^2 + 7x \), write it in standard form and state the value of the leading coefficient.

(d) If \( f(x) = 2x^2 - 3x + 1 \), then find, without using your calculator, the value of \( f(-2) \). What point must lie on this quadratic’s graph based on this calculation?

Quadratics still behave in similar ways to other functions. Inputs go in, outputs come out. But, they start to behave differently from linear and exponential functions because sometimes outputs repeat for quadratics.

**Exercise #2:** Consider the simplest of all quadratic functions, \( f(x) = x^2 \).

(a) Fill out the table below without using your calculator.

<table>
<thead>
<tr>
<th>( x )</th>
<th>-3</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y = x^2 )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(b) Graph the function on the grid shown.

(c) What is the range of this quadratic function?
Quadratic functions can obviously be more complicated than our last example, but, strangely enough, they all have the same general shape, which is known as a **parabola**. Let’s explore the next quadratic function with the help of technology. We will also introduce some important terminology.

**Exercise #3:** Consider the quadratic function \( y = x^2 - 2x - 8 \).

(a) Using your calculator to help generate a table, graph this parabola on the grid given. Show a table of values that you use to create the plot.

(b) State the **range** of this function.

(c) Over what **domain interval** is the function **increasing**?

(d) State the coordinates of the parabola’s turning point (also known as its vertex and its minimum point).

(e) Draw the axis of symmetry of the parabola and write its equation below and on the graph.

(f) What are the \( x \)-intercepts of this function? These are also known as the function’s **zeroes**. Why does this name make sense? As a suggestion, write out their full \( xy \)-pair coordinates.

**Exercise #4:** The quadratic function \( f(x) \) has selected values shown in the table below.

(a) What are the coordinates of the turning point?

(b) What is the range of the quadratic function?

\[
\begin{array}{|c|c|}
\hline
x & f(x) \\
\hline
-1 & 4 \\
0 & 9 \\
1 & 12 \\
2 & 13 \\
3 & 12 \\
4 & 9 \\
5 & 4 \\
6 & -2 \\
\hline
\end{array}
\]
INTRODUCTION TO QUADRATIC FUNCTIONS
COMMON CORE ALGEBRA I HOMEWORK

FLUENCY

1. Which of the following is a quadratic function?
   (1) \( y = 3x - 2 \)  (3) \( y = x^2 - 4 \)
   (2) \( y = x^3 + 2x^2 - 1 \)  (4) \( y = 6(2)^x \)

2. The quadratic function \( y = 9 - x^2 + 4x \) written in standard form would be
   (1) \( y = -x^2 + 4x + 9 \)  (3) \( y = x^2 - 4x + 9 \)
   (2) \( y = x^2 - 9x + 4 \)  (4) \( y = -x^2 - 4x + 9 \)

3. Which of the following would be the leading coefficient of \( f(x) = 6 - x + 7x^2 \)?
   (1) \(-1\)  (3) \(7\)
   (2) \(6\)  (4) \(-7\)

4. Which of the following points lies on the graph of \( y = x^2 - 5 \)?
   (1) \((3, -2)\)  (3) \((5, 0)\)
   (2) \((-2, -1)\)  (4) \((-1, -6)\)

5. A quadratic function is partially given in the table below. Which of the following are the coordinates of its turning point?
   (1) \((0, 6)\)  (3) \((3, 15)\)
   (2) \((10, 2)\)  (4) \((7, -1)\)

<table>
<thead>
<tr>
<th>(x)</th>
<th>(-2)</th>
<th>(-1)</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>(y)</td>
<td>10</td>
<td>7</td>
<td>6</td>
<td>7</td>
<td>10</td>
<td>15</td>
</tr>
</tbody>
</table>

6. Given the quadratic function shown below whose turning point is \((2, -4)\), which of the following gives the domain interval over which this function is decreasing?
   (1) \(x > -4\)  (3) \(x > 2\)
   (2) \(x < -4\)  (4) \(x < 2\)
7. Consider the function \( f(x) = x^2 + 2x - 3 \).

(a) Using your calculator, create an accurate graph of \( f(x) \) on the grid provided.

(b) State the coordinates of the turning point of \( f(x) \). Is this point a maximum or minimum?

(c) State the range of this quadratic function.

(d) State the zeroes of this function (the \( x \)-intercepts).

(e) Over what interval is this function negative? In other words, over what \( x \)-values is the output (or \( y \)-value) to this function negative?

(f) Over what interval is this function increasing?

8. A quadratic function \( g(x) \) is shown partially in the table below. The turning point of the function has the coordinates \((3, -8)\). Think about how outputs repeat in a quadratic function and answer the following.

<table>
<thead>
<tr>
<th>( x )</th>
<th>(-1)</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>( g(x) )</td>
<td>24</td>
<td>0</td>
<td>-6</td>
<td>-8</td>
<td></td>
<td>10</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(a) Fill in the missing output values from the table. (b) What are the zeroes of the function?

(c) What is this function’s \( y \)-intercept? (d) For the domain interval \(-1 \leq x \leq 7\), what is the range of the function?
MORE WORK WITH PARABOLAS
COMMON CORE ALGEBRA I

The graphs of quadratic functions are more complex than linear and exponential because they include a **turning point** that is either the location of a **maximum** or a **minimum**. Today we will explore these functions more by using our calculator technology. But first, we need to examine one additional quadratic function by hand.

**Exercise #1:** Consider the simple quadratic function \( y = -x^2 \).

(a) Write this parabola in the form \( y = ax^2 \), where \( a \) is the leading coefficient. Then, fill out the table below.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y = -x^2 )</th>
<th>( (x, y) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>-2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>-1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(b) Graph the parabola given in this table on the grid provided. What is the range of this quadratic?

Some parabolas are concave up (open upward) and some are concave down (open downward). Let’s see if we can find a pattern that tells us what controls this behavior.

**Exercise #2:** Use your graphing calculator with a **STANDARD WINDOW** to sketch each of the following.

\[
y = 3x^2 + 6x - 4 \\
y = -x^2 + 6x + 1 \\
y = -2x^2 - 8x - 4
\]
We will explore the reason for this pattern more in the next exercise with much simpler quadratic functions.

**Exercise #3:** Use your calculator to sketch a graph of each of the following quadratics using the indicated window.

(a) \( y = 2x^2 \)  
(b) \( y = 3x^2 \)  
(c) \( y = 4x^2 \)  

(d) \( y = -2x^2 \)  
(e) \( y = -3x^2 \)  
(f) \( y = -4x^2 \)  

So, it appears that we can now determine what controls the direction a parabola opens.

**Exercise #4:** For the quadratic \( y = ax^2 + bx + c \) fill in the blanks:

1. The parabola will **open upwards**, in other words look like \( \uparrow \) if \( a > 0 \).  
   
   This type of quadratic function will have a **minimum y-value**.

2. The parabola will **open downwards**, in other words look like \( \downarrow \) if \( a < 0 \).  
   
   This type of quadratic function will have a **maximum y-value**.
MORE WORK WITH PARABOLAS
COMMON CORE ALGEBRA I HOMEWORK

FLUENCY

1. Which of the following could be the equation of the quadratic shown below? Explain your reasoning.

   (1) \( y = -3x^2 + 8x - 5 \)
   (2) \( y = 4x^2 - 6x + 7 \)
   (3) \( y = -2x^2 + 12x + 11 \)
   (4) \( y = x^2 - 8x - 2 \)

   Reasoning:

2. Based on the quadratic function shown in the table below, which of the following is the range of this function?

   (1) \( y \geq -7 \)
   (2) \( y \geq 3 \)
   (3) \( y \leq 4 \)
   (4) \( y \leq 11 \)

<table>
<thead>
<tr>
<th>x</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>3</td>
<td>9</td>
<td>11</td>
<td>9</td>
<td>3</td>
<td>-7</td>
</tr>
</tbody>
</table>

For Problems 3 – 5, use tables on your calculator to help you investigate these functions.

3. Which of the following quadratics will have a maximum value at \( x = 3 \)?

   (1) \( y = x^2 - 6x + 19 \)
   (2) \( y = -4x^2 + 24x - 21 \)
   (3) \( y = -2x^2 + 20x - 49 \)
   (4) \( y = 2x^2 - 3x + 7 \)

4. Which of the following quadratics will have a minimum value of \(-5\) at \( x = 7 \)?

   (1) \( y = x^2 - 14x + 39 \)
   (2) \( y = -x^2 + 14x - 54 \)
   (3) \( y = x^2 - 14x + 44 \)
   (4) \( y = -x^2 - 10x - 18 \)

5. The parabola \( y = -x^2 + 12x - 11 \) has an axis of symmetry of \( x = 6 \). Which of the following represents its range?

   (1) \( y \geq -11 \)
   (2) \( y \leq 25 \)
   (3) \( y \leq 6 \)
   (4) \( y \geq 10 \)
APPLICATIONS

6. The height of an object that is traveling through the air can be well modeled by a quadratic function that opens downward. An object is fired upward and its height in feet above the ground is given by:

\[ h(t) = -16t^2 + 64t + 80 \]

where the input, \( t \), is the time, in seconds, the object has been in the air

(a) Using your calculator, sketch a graph of the object’s height for all times where it is at or above the ground.

(b) What is its maximum height in feet?

(c) At what time does it hit the ground?

(d) Over what time interval is its height increasing?

7. The cost per computer produced at a factory depends on how many computers the factory produces in a day. The cost function is modeled by

\[ C(n) = \frac{1}{500}n^2 - n + 200 \]

where \( n \) is the number of computers produced in a day and \( C(n) \) is the unit cost, in dollars per computer.

(a) Calculate \( C(50) \) and give an interpretation of your answer in terms of the scenario described.

(b) Does the cost have a minimum or maximum value? Explain. Use your calculator to find it.

(c) Based on (b), can this function have any real zeroes? Explain your thought process.
THE SHIFTED FORM OF A PARABOLA
COMMON CORE ALGEBRA I

Although the standard form of a parabola has advantages for certain applications, it is not helpful locating the most important point on the parabola, the turning point. In this lesson, we will learn about a form of a parabola where the turning point is fairly obvious. First, though, a review of simple parabolas.

Exercise #1: Without using your calculator, sketch each of the parabolas shown below on your own set of axes. State the coordinates of the turning point of both.

(a) \( y = 2x^2 \)

(b) \( y = -3x^2 \)

(c) If we have a parabola in the form \( y = ax^2 \) then it has a turning point at ________________.

Now we would like to try to develop a pattern to see how a function can have its graph shifted.

Exercise #2: Consider the basic quadratic function \( f(x) = x^2 \) and the more complex quadratic function \( g(x) = (x-2)^2 - 4 \). The graph of \( f(x) = x^2 \) is shown on the grid already.

(a) Using your calculator to generate a table, sketch a graph of \( g \).

(b) How would you need to shift the graph of \( f(x) \) to get the graph of \( g(x) \)?

(c) What is the turning point of \( g(x) \)? Where do you “see” the turning point in the function’s equation?
Let’s keep looking at this pattern but more simply.

**Exercise #3:** The parabola $y = x^2$ is again shown on the grid below. Consider the quadratic functions $y = x^2 + 2$ and $y = x^2 - 4$.

(a) Using your calculator to generate tables, sketch these two quadratics and label.  
(b) What was the effect of adding a constant to the overall function?

(c) State the coordinates of the turning points of each of the parabola you drew in (a).

\[ y = x^2 + 2 \quad \text{and} \quad y = x^2 - 4 \]

(d) What would the coordinates of the turning point of the parabola $y = x^2 - 150$ be?

Now let’s see about that number added and subtracted from the input variable, $x$, before it is even squared.

**Exercise #4:** Yet (again), the parabola $y = x^2$ is graphed below. Now consider $y = (x + 3)^2$ and $y = (x - 1)^2$.

(a) Using your calculator to generate tables, sketch these two quadratics and label.  
(b) Why is the horizontal shift counterintuitive?

(c) State the coordinates of the turning points of each of the parabola you drew in (a).

\[ y = (x + 3)^2 \quad \text{and} \quad y = (x - 1)^2 \]

(d) Determine the coordinate of the turning points of each of the following quadratics. Note that the value of $a$ is irrelevant.

\[ y = (x - 8)^2 + 5 \quad \text{and} \quad y = 5(x + 1)^2 - 4 \quad \text{and} \quad y = -2(x - 3)^2 - 10 \]
1. The grid below shows the graph of $y = x^2$ with particular points emphasized. On the same grid, draw the following quadratic functions. Try to do these as best as possible without using your calculator and then check your answers. Label each with its letter or equation.

(a) $y = x^2 - 6$

(b) $y = x^2 + 1$

(c) $y = (x+3)^2$

(d) $y = (x-4)^2$

2. Again, the function $y = x^2$ is shown below. Graph each of the following more complicated quadratics without the use of your calculator. Then, use it to check that you have shifted the correct amounts. Label each with its letter or equation.

(a) $y = (x-1)^2 - 4$

(b) $y = (x+3)^2 - 1$

3. Which of the following equations represents the graph shown below given that it is a shift of the function $y = x^2$. Explain your choice.

(1) $y = (x-3)^2 - 4$

(2) $y = (x+3)^2 - 4$

(3) $y = (x-3)^2 + 4$

(4) $y = (x+3)^2 + 4$
4. State the turning points for each of the following quadratic functions and state whether the parabola opens upwards or downwards. Remember, the direction is opens only depends on the leading coefficient.

(a) \( y = 4(x-2)^2 + 7 \)  
(b) \( y = -3(x+6)^2 + 4 \)  
(c) \( y = -(x+4)^2 - 3 \)  

(d) \( y = \frac{1}{2}(x+1)^2 - 7 \)  
(e) \( y = 9 - x^2 \)  
(f) \( y = -16(x-5)^2 + 11 \)  

APPLICATIONS

5. An object traveling under an acceleration due to gravity alone will have a height, \( h \), in meters above the ground \( t \)-seconds after it was fired given by

\[ h = -4.9(t-6)^2 + 210 \]

(a) At what height does the object begin at \( t = 0 \)?
Show work that supports your answer.

(b) What is the peak height this object reaches in meters? When does it reach this height, in seconds?

(c) Although you should be able to answer part (b) without your calculator, provide evidence in a table form that supports your answer from (b).

(d) Using your calculator, sketch a graph of the height over the interval shown. Label your answers from (a) and (b).

REASONING

6. Why does the turning point of the quadratic \( y = a(x-h)^2 + k \) not depend on the value of \( a \). In other words why do both \( y = 5(x-2)^2 + 3 \) and \( y = -8(x-2)^2 + 3 \) have turning points of \( (2, 3) \)?
COMPLETING THE SQUARE
COMMON CORE ALGEBRA I

The turning point of a parabola and its general shape are relatively easy to determine if the quadratic function is written in its **shifted or vertex form**. Review this in the first exercise.

**Exercise #1:** Given the function \( y = (x - 3)^2 + 2 \) do the following.

(a) Give the coordinates of the turning point.  
(b) Determine the range by drawing a rough sketch.

The question then is how we take a quadratic of the form \( y = ax^2 + bx + c \) and put it into its shifted form. This procedure is known as **Completing the Square**. But, it needs some additional review.

**Exercise #2:** Write each of the following as an equivalent trinomial.

(a) \((x + 5)^2\)  
(b) \((x - 1)^2\)  
(c) \((x + 4)^2\)

**Exercise #3:** Given each trinomial in Exercise #2 of the form \( x^2 + bx + c \), what is true about the relationship between the value of \( b \) and the value of \( c \)? Illustrate.

**Exercise #4:** Each of the following trinomials is a perfect square. Write it in factored (or perfect square) form.

(a) \( x^2 + 20x + 100 \)  
(b) \( x^2 - 6x + 9 \)  
(c) \( x^2 + 2x + 1 \)
We are finally ready to learn the method of **Completing the Square**. This method has many uses, but the one we will work with today is to manipulate equations of quadratics from their **standard form** to their **vertex form**.

**Exercise #5:** The quadratic \( y = x^2 - 4x - 1 \) is shown graphed below.

(a) Consider only the binomial \( x^2 - 4x \). What would you need to add on to it to create a perfect square trinomial? (See Exercise #3).

(b) In order to add zero to the binomial \( x^2 - 4x \), what should we subtract to offset adding 4 to make it a perfect square?

(c) Use the Method of Completing the Square now to rewrite the trinomial \( x^2 - 4x - 1 \) in an equivalent, shifted form. According to this form, what are the coordinates of the vertex? Verify by examining the graph.

This procedure is what is known as an **algorithm**. In other words, we follow a recipe. Here it is:

**Completing the Square**

For the quadratic \( y = x^2 + bx + c \) (note that \( a = 1 \)).

1. Find half of the value of \( b \), i.e. \( \frac{b}{2} \)

2. Square it, i.e. \( \left( \frac{b}{2} \right)^2 \)

3. Add and subtract it

There is nothing like practice on these.

**Exercise #6:** Write each quadratic in vertex form by Completing the Square. Then, identify the quadratic’s turning point. The last two problems will involve fractions. Stick with it!

(a) \( y = x^2 + 6x - 2 \)  
(b) \( y = x^2 - 2x + 11 \)  
(c) \( y = x^2 - 10x + 27 \)

(d) \( y = x^2 + 8x \)  
(e) \( y = x^2 + 5x + 4 \)  
(f) \( y = x^2 - 9x - 2 \)
COMPLETING THE SQUARE
COMMON CORE ALGEBRA I HOMEWORK

FLUENCY

1. Find each of the following products in standard form.
   (a) \((x + 4)^2\)  
   (b) \((x - 1)^2\)  
   (c) \((x + 8)^2\)

   (d) \((x - 7)^2\)  
   (e) \((x + 2)^2\)  
   (f) \((x - 10)^2\)

2. Each of the following trinomials is a perfect square. Write it in factored form, i.e. \((x + a)^2\) or \((x - a)^2\).
   (a) \(x^2 + 6x + 9\)  
   (b) \(x^2 - 22x + 121\)  
   (c) \(x^2 + 10x + 25\)

   (d) \(x^2 + 30x + 225\)  
   (e) \(x^2 - 2x + 1\)  
   (f) \(x^2 - 18x + 81\)

3. Place each of the following quadratic functions, written in standard form, into vertex form by completing the square. Then, identify the coordinates of its turning point.
   (a) \(y = x^2 - 12x + 40\)  
   (b) \(y = x^2 + 4x + 14\)  
   (c) \(y = x^2 - 24x + 146\)
APPLICATIONS

4. A cable is attached at the same height from two poles and hangs between them such that its height above the ground, \( y \), in inches, can be modeled using the equation:

\[
y = x^2 - 16x + 67
\]

where \( x \) represents the horizontal distance from the left pole, in feet.

(a) What height is point A above the ground? Show your work and use proper units.

(b) Write the equation in vertex form.

(c) What is the difference in the heights of points A and B? Show your analysis and include units.

(d) What is the horizontal distance that separates points A and C? Explain your reasoning.

REASONING

5. Use the vertex form of each of the following quadratic functions to determine which has the lowest \( y \)-value.

\[
y = x^2 - 8x + 6 \quad \quad \quad y = x^2 + 6x + 1
\]

6. Two quadratic functions are shown below, \( f(x) \) and \( g(x) \). Determine which has the lower minimum value. Explain how you arrived at your answer.

\[
f(x) = x^2 + 10x
\]

\[
g(x) = \begin{array}{ccccccccc}
x & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\
-9 & -14 & -17 & -18 & -17 & -14 & -9 \\
\end{array}
\]
STRETCHING PARABOLAS AND MORE COMPLETING THE SQUARE
COMMON CORE ALGEBRA I

We want to do one additional lesson to make sure we fully appreciate what the leading coefficient in a parabola tells us about it. Plus, we will get a bit more work with Completing the Square; although, it will be more challenging.

Exercise #1: Let’s understand what \( a \) really does in \( y = ax^2 + bx + c \). So far we know that if \( a \) is positive, the parabola opens upwards and if \( a \) is negative, it opens downward. Let’s see if we can deepen our understanding.

(a) Using your calculator, sketch a graph of \( y = x^2 \), \( y = 2x^2 \), and \( y = 4x^2 \) on the axes below. Use the window indicated on the axes. Label each with its equation.

(b) Explain what is happening when we multiply \( x^2 \) by \( a \).

So, \( a \) stretches and compresses a parabola depending on whether it is greater than 1 or between 0 and 1. Sound familiar? This is similar to the base of an exponential function.

Exercise #2: The graph below shows the curves that are listed. Write the number of the equation of each beside its curve.

1. \( y = 3x^2 \)
2. \( y = \frac{1}{2}x^2 \)
3. \( y = -2x^2 \)
4. \( y = x^2 \)
5. \( y = -x^2 \)
Since all quadratics of the form \( y = ax^2 \) have their turning points at the origin we can also identify turning points if we can place a quadratic in vertex form even if \( a \) doesn’t equal 1. This is harder, mechanically.

**Exercise #3:** Consider the quadratic \( y = 2x^2 - 12x + 11 \).

(a) Use the Method of Completing the Square to write this equation in the vertex form \( y = a(x - h)^2 + k \).

(b) What are the coordinates of the turning point of this quadratic based on (a)? Provide evidence from a calculator table that supports this answer.

Completing the Square, when the leading coefficient doesn’t equal 1, is much more difficult to master and to understand. Always remember that you are writing an equivalent expression by essentially adding zero in one way or another.

**Exercise #4:** Use the Method of Completing the Square to write each of the following quadratic functions in the vertex form \( y = a(x - h)^2 + k \). Identify the turning point of the quadratic from this form. State whether it is a maximum or minimum.

(a) \( y = 5x^2 + 20x + 23 \)  
(b) \( y = -2x^2 + 4x + 7 \)

(c) \( y = 6x^2 - 24x + 14 \)  
(d) \( y = -x^2 - 12x - 33 \)
STRETCHING PARABOLAS AND MORE COMPLETING THE SQUARE
COMMON CORE ALGEBRA I HOMEWORK

FLUENCY

1. The graph to the right contains the following functions. Match their letters to the correct curves.

(A) \( y = x^2 \)
(B) \( y = -2x^2 \)
(C) \( y = \frac{1}{4}x^2 \)
(D) \( y = -x^2 \)
(E) \( y = 3x^2 \)

2. Which of the following equations models the graph shown at the right? Explain how you made your choice?

(1) \( y = (x-1)^2 - 5 \)
(2) \( y = -3(x+1)^2 - 5 \)
(3) \( y = (x+1)^2 - 5 \)
(4) \( y = 2(x-1)^2 - 5 \)

3. Use the method completing the square to write each of the following quadratic functions in the form \( y = a(x-h)^2 + k \). Then, identify the turning point and whether it is a maximum or minimum.

(a) \( y = 3x^2 - 12x + 17 \)
(b) \( y = -5x^2 + 40x - 70 \)
APPLICATIONS

4. The vertical height of projectiles above level ground can be modeled by equations in the form:

\[ h(t) = -16(t-t_{\text{max}})^2 + h_{\text{max}} \]

where \( h_{\text{max}} \) is the maximum height in feet and \( t_{\text{max}} \) is the time, in seconds, when it occurs.

(a) A given projectile has a height function given by \( h(t) = -16(t-8)^2 +156 \). What is its maximum height and at what time, \( t \), does it occur?

(b) A projectile has a height function given by \( h(t) = -16t^2 +160t +120 \). Write this in the form shown above (vertex form).

(c) What is the maximum height and at what time does it occur for the projectile from (b)?

(d) At what height does the projectile in (b) start above the ground? Show the work that leads to your answer.

REASONING

5. Every quadratic function can be placed in a vertex form: \( y = a(x-h)^2 + k \). If we know the turning point of the parabola and one other point we can uniquely find this equation. Let’s say we want to find the equation of a parabola that has a turning point at \((3,9)\) and passes through the point \((5,29)\).

(a) Write the equation of this parabola in the form \( y = a(x-h)^2 + k \), leaving \( a \) as an unknown constant or parameter.

(b) Substitute the point \((5,29)\) into the equation from part (a) and find the value of \( a \).

(c) State the final equation of this parabola in vertex form and verify that it has the correct turning point and passes through \((5,29)\) by examining a table on your calculator.
THE ZEROS OF A QUADRATIC
COMMON CORE ALGEBRA I

The x-locations on any function where the output (the y-coordinate) is equal to zero are known, not surprisingly, as the zeroes of the function. These are amazingly important in applied settings. When they are rational numbers then they can be found using a factoring technique. We’ll develop the idea in the first exercise.

Exercise #1: Consider the quadratic function \( y = x^2 - 2x - 3 \). It’s graph is shown below.

(a) What are the zeroes of the function? Write their x-values and circle them on the graph.

(b) Verify that the positive zero is correct by showing that \( y = 0 \).

(c) Factor the expression \( x^2 - 2x - 3 \). How do these factors compare to the zeroes?

(d) Based on (c), determine where the zeroes of \( y = x^2 + 3x - 10 \) are algebraically. Verify using a table.

What is really going on here is perhaps the second most important equation solving technique, known as the Zero Product Law.

### THE ZERO PRODUCT LAW

If two or more quantities have a product of zero then at least one of them must be equal to zero. In symbolic form:

\[
\text{If } a \cdot b = 0 \text{ then either } a = 0 \text{ or } b = 0 \text{ (or both are zero)}
\]

Exercise #2: Use the Zero Product Law to find all solutions to each of the following equations.

(a) \((x + 7)(x - 2) = 0\)

(b) \((2x - 1)(3x + 4) = 0\)
The **Zero Product Law** is remarkable because it allows us to solve equations with an $x^2$ or higher level term in it, as long as the expression set equal to zero can be factored.

**Exercise #3:** Find the roots (solutions) to each of the following equations by using the **Zero Product Law**. Sometimes you will be instructed to solve by factoring.

(a) $x^2 + 4x - 12 = 0$

(b) $2x^2 - 14x = 0$

(c) $x^2 - 25 = 0$

(d) $2x^2 + 5x - 12 = 0$

**Exercise #4:** Find the zeroes of the quadratic function $y = 3x^2 - 6x - 24$ algebraically. Then verify your answer by using your calculator to sketch a graph of the parabola using the window indicated on the axes below. Clearly mark the zeroes on the graph.
THE ZEROS OF A QUADRATIC
COMMON CORE ALGEBRA I HOMEWORK

FLUENCY

1. The roots of \( x^2 - 6x - 16 = 0 \) can be found by factoring as

   (1) \([-16, 6]\)  
   (2) \([-8, 2]\)  
   (3) \([-2, 8]\)  
   (4) \([6, 16]\)

2. The equation \((2x - 3)(x + 7) = 0\) has a solution set of

   (1) \([-7, 1\frac{1}{2}]\)  
   (2) \([3, 7]\)  
   (3) \([-7, 3]\)  
   (4) \([\frac{1}{2}, -3]\)

3. Find the roots of each of the following equations by factoring:

   (a) \( x^2 - 36 = 0 \)  
   (b) \( x^2 + 12x + 27 = 0 \)  
   (c) \( 3x^2 + 5x - 2 = 0 \)  
   (d) \( 20x^2 - 10x = 0 \)  
   (e) \( 10x^2 + x - 21 = 0 \)  
   (f) \( 4x^2 - 16x - 84 = 0 \)
4. Consider the quadratic function \( y = x^2 - 4x - 5 \).

(a) Using your calculator, graph the function on the grid provided.

(b) State the zeroes of the function by inspecting the graph. Circle their locations.

(c) Find the zeroes algebraically by factoring. Verify that your answers match (b).

APPLICATIONS

5. A baking soda rocket is fired upwards with an initial speed of 80 feet per second. Its height, \( h \), above the ground in feet can be modeled using the equation:

\[
h(t) = -16t^2 + 80t\]

where \( t \) is the time since launch in seconds

At what time, \( t > 0 \), does the rocket hit the ground? Find algebraically using factoring.

REASONING

6. The two quadratic equations below have the same solutions. Can you determine why? Completely factor both to see what they have in common.

\[
x^2 - 7x + 12 = 0 \\
3x^2 - 21x + 36 = 0
\]
MORE ZERO PRODUCT LAW
COMMON CORE ALGEBRA I

The Zero Product Law’s importance to mathematics cannot be overstated. It finally allows us, in certain situations, to solve equations that are higher-order polynomials than just linear. Of course, for it to work, we must have two conditions met: (1) we must have the equation set equal to zero and (2) we must be able to factor the expression equal to zero.

**Exercise #1:** Solve each of the following equations using factoring.

(a) \( x^2 + 2x - 35 = 0 \)

(b) \( 3x^2 - 30x + 48 = 0 \)

(c) \((x - 3)(x+1) + (x - 3)(2x - 7) = 0\)

Let’s remember why this is such a crucial skill in terms of parabolas.

**Exercise #2:** James graphed the quadratic \( y = 3x^2 + x - 10 \) using tables on his calculator and found the graph shown below. He can tell from his graph and table that \( x = -2 \) is one of the two zeroes. But, he couldn’t tell what the other was because it did not fall on an integer location (circled).

(a) Write down an equation that would allow you to solve for the zeroes of this function.

(b) How does knowing that \( x = -2 \) is a zero help you factor the trinomial \( 3x^2 + x - 10 \)? Factor it.

(c) Solve the equation in (a) using factoring to find the other zero of this function.
We can even explore higher-order polynomials and their zeroes on a very limited basis. So far the best we have done is an $x^2$, but polynomials that contain an $x^3$ can also be analyzed. These are known as **cubics**.

**Exercise #3:** Consider the cubic function $f(x) = x^3 - 9x$.

(a) Find the zeroes of this function algebraically by factoring. 

(b) Use your calculator to sketch a graph of this function. Circle the zeroes on the graph.

You will study higher-order polynomial functions in Algebra II. But, you should be able to find the zeroes for a limited number of **cubic polynomials** that can be easily factored. In our last exercise, we’d like to explore the relationship between the **zeroes of a quadratic** and the $x$-coordinate of its turning point.

**Exercise #4:** Consider the quadratic $y = x^2 - 8x + 15$.

(a) Find the zeroes of this function algebraically using factoring. 

(b) Write the quadratic function in vertex form and identify the coordinates of its turning point.

(c) What is true about the $x$-coordinate of the turning point compared to the zeroes you found in (a)?

(d) Without using a calculator, sketch a graph of this quadratic on the axes below.

**Exercise #5:** A quadratic function can be written in factored form as $y = (x+3)(x-7)$. Which of the following would be the $x$-coordinate of its turning point?

1. $x = 6$ 
2. $x = 2$ 
3. $x = 5$ 
4. $x = 4$
MORE ZERO PRODUCT LAW WORK
COMMON CORE ALGEBRA I HOMEWORK

FLUENCY

1. Solve each of the following:
   (a) \((3x + 1)(x - 2) = 0\)
   (b) \(5(x - 3)(x + 8) = 0\)

2. Solve each of the following by factoring:
   (a) \(2x^2 - 19x + 35 = 0\)
   (b) \(4x^2 - 52x + 120 = 0\)

3. Solve each of the following by factoring:
   (a) \(30x^2 - 80x = 0\)
   (b) \(x^2 - x = 0\)

4. Solve each of the following by factoring a binomial gcf out of each term:
   (a) \((2x - 1)(x + 5) + (2x - 1)(x - 2) = 0\)
   (b) \((x - 8)(5x + 4) - (x - 8)(2x + 6) = 0\)
5. Consider the cubic polynomial \( y = x^3 + 2x^2 - 8x \).

   (a) Find the **three** zeroes of this function algebraically by factoring.

   (b) Use your calculator to sketch a graph of the cubic on the axes below. Mark your answers from (a).

   

   ![Graph of the cubic function](image)

**REASONING**

6. Consider the quadratic function \( y = x^2 + 4x - 5 \).

   (a) Find its zeroes algebraically.

   (b) Using your calculator, sketch a graph of the function on the axes given.

   (c) Find the zeroes of \( y = 2x^2 + 8x - 10 \) algebraically.

   (d) Using your calculator, sketch a graph of this function on the same axes. How does the second graph compared the first that you drew?
QUADRATIC WORD PROBLEMS
COMMON CORE ALGEBRA I

Now that we have the Zero Product rule as a method for solving quadratic equations that are **factorable when set equal to zero**, we can also model scenarios that are quadratic in nature and solve them for rational solutions with factoring.

**Exercise #1:** Consider a rectangle whose area is 45 square feet. If we know that the length is one less than twice the width, then we would like to find the dimensions of the rectangle.

(a) If we represent the width of the rectangle using the variable \( W \), then write an expression for the length of the rectangle, \( L \), in terms of \( W \).

(b) Set up an equation that could be used to solve for the width, \( W \), based on the area.

(c) Solve the equation to find both dimensions. Why is one of the solutions for \( W \) not viable?

**Exercise #2:** A square has one side increased in length by two inches and an adjacent side decreased in length by two inches. If the resulting rectangle has an area of 60 square inches, what was the area of the original square? First, draw some possible squares and rectangles to see if you can solve by guess-and-check. Then, solve it algebraically.
We can certainly play around with word problems that involve strictly numbers. For example…

**Exercise #3:** There are two rational numbers that have the following property: when the product of seven less than three times the number with one more than the number is found it is equal to two less than ten times the number. Find the two rational numbers that fit this description.

And, of course, who can forget our work with **consecutive integers** from the linear unit?

**Exercise #4:** Find all sets of two consecutive integers such that their product is eight less than ten times the smaller integer.

**Exercise #5:** Brendon claims that the number five has the property that the product of three less than it with one more than it is the same as the three times one less than it. Show that Brendon’s claim is true and algebraically find the other number for which this is true.
QUADRATIC WORD PROBLEMS
COMMON CORE ALGEBRA I HOMEWORK

FLUENCY

1. The product of two consecutive positive even integers is 14 more than their sum. Set up an equation that can used to find the two numbers and solve it.

2. The length of a rectangle is 4 less than twice the width. The area of the rectangle is 70 square feet. Find the width, \( w \), of the rectangle algebraically. Explain why one of the solutions for \( w \) is not viable.

3. Two sets of three consecutive integers have a property that the product of the larger two is one less than seven times the smallest. Set up and solve an equation that can be used to find both sets of integers.
APPLICATIONS

4. A curious pattern occurs in a group of people who all shake hands with one another. It turns out that you can predict the number of handshakes that will occur if you know the number of people.

If we are in a room of 5 people, we can determine the number of handshakes by this line of reasoning:

The first person will shake 4 hands (she won’t shake her own). The second person will shake 3 hands (he won’t shake his own of the hand of the first person, they already shook). The third person will shake 2 hands (same reasoning). The fourth person will shake 1 hand (that of the fifth person). The fifth person will shake 0 hands. So there will be a total of 1+2+3+4=10 handshakes.

(a) Determine the number of handshakes, \( h \), that will occur for each number of people, \( n \), in a particular room.

<table>
<thead>
<tr>
<th>( n ) (people)</th>
<th>Calculation</th>
<th>( h ) (handshakes)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>1+2+3+4=10</td>
<td>10</td>
</tr>
<tr>
<td>6</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(b) Using knowledge from Algebra II, Prestel proposes the formula \( h = \frac{n(n-1)}{2} \) to find the number of handshakes, \( h \), if he knows the number of people. Test the formula and compare with the results you found in (a).

<table>
<thead>
<tr>
<th>( n ) (people)</th>
<th>( h = \frac{n(n-1)}{2} )</th>
<th>Comparison to (a)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td></td>
<td></td>
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<tr>
<td>3</td>
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<td>4</td>
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<td>5</td>
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<tr>
<td>6</td>
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</tbody>
</table>

(c) Assuming Prestel’s formula is correct, algebraically determine number of people in a room if there are 66 handshakes that occur.
UNIT #9

ROOTS AND IRRATIONAL NUMBERS

Lesson #1 – Square Roots
Lesson #2 – Irrational Numbers
Lesson #3 – Square Root Functions and Shifting
Lesson #4 – Solving Quadratics Using Inverse Operations
Lesson #5 – Finding Zeroes by Completing the Square
Lesson #6 – The Quadratic Formula
Lesson #7 – Final Work with Quadratic Equations
Lesson #8 – Cube Roots
Square roots, cube roots, and higher level roots are important mathematical tools because they are the **inverse operations** to the operations of **squaring and cubing**. In this unit we will study these operations, as well as numbers that come from using them. First, some basic review of what you’ve seen before.

**Exercise #1:** Find the value of each of the following **principal square roots**. Write a reason for your answer in terms of a multiplication equation.

(a) $\sqrt{25}$ (b) $\sqrt{9}$ (c) $\sqrt{100}$

(d) $\sqrt{0}$ (e) $\sqrt{\frac{1}{4}}$ (f) $\sqrt{\frac{64}{9}}$

It is generally agreed upon that all **positive, real numbers** have two square roots, a positive one and a negative one. We simply designate which one we want by either including a negative sign or leaving it off.

**Exercise #2:** Give all square roots of each of the following numbers.

(a) 4 (b) 36 (c) $\frac{1}{16}$

**Exercise #3:** Given the function \( f(x) = \sqrt{x + 3} \), which of the following is the value of \( f(46) \)?

(1) 22 (3) 16

(2) 5 (4) 7

Square roots have an interesting property when it comes to multiplication. We will discover that property in the next exercise.

**Exercise #4:** Find the value of each of the following products.

(a) $\sqrt{4} \cdot \sqrt{9} =$ (b) $\sqrt{4} \cdot 9 =$

(c) $\sqrt{4} \cdot \sqrt{25} =$ (d) $\sqrt{4} \cdot 25 =$
What you should notice in the last exercise is the following important property of square roots.

**MULTIPLICATION PROPERTY OF SQUARE ROOTS**

1. \( \sqrt{a} \cdot \sqrt{b} = \sqrt{a \cdot b} \)

   likewise  

2. \( \sqrt{a} \cdot \sqrt{b} = \sqrt{a} \cdot \sqrt{b} \)

One obvious use for this is to multiply two “unfriendly” square roots to get a nice result.

**Exercise #5:** Find the result of each of the following products.

(a) \( \sqrt{2} \cdot \sqrt{8} = \)

(b) \( \sqrt{12} \cdot \sqrt{3} = \)

(c) \( \sqrt{20} \cdot \sqrt{5} = \)

One less obvious use for the square root property above is in simplifying square roots of non-perfect squares. This is a fairly antiquated skill that is almost completely irrelevant to algebra, but it often arises on standardized tests and thus is a good skill to become fluent with.

**Exercise #6:** To introduce simplifying square roots, let’s do the following first.

(a) List out the first 10 perfect squares (starting with 1).

(b) Now consider \( \sqrt{18} \). Which of these perfect squares is a factor (divides) of 18?

(c) Simplify the \( \sqrt{18} \). This is known as writing it in simplest radical form.

The key to simplifying any square root is to find the largest perfect square that is a factor of the radicand (the number under the square root).

**Exercise #7:** Write each of the following square roots in simplest radical form.

(a) \( \sqrt{8} \)

(b) \( \sqrt{45} \)

(c) \( \sqrt{48} \)

(d) \( -\sqrt{75} \)

(e) \( \sqrt{72} \)

(f) \( -\sqrt{500} \)
SQUARE ROOTS
COMMON CORE ALGEBRA I HOMEWORK

FLUENCY

1. Simplify each of the following. Each will result in a rational number answer. You can check your work using your calculator, but should try to do all of them without it.

   (a) \( \sqrt{36} \)  
   (b) \( -\sqrt{4} \)  
   (c) \( \sqrt{121} \)  
   (d) \( \frac{1}{\sqrt{9}} \)

   (e) \( -\sqrt{100} \)  
   (f) \( \frac{81}{\sqrt{36}} \)  
   (g) \( -\frac{1}{\sqrt{16}} \)  
   (h) \( -\sqrt{144} \)

2. Find the final, simplified answer to each of the following by evaluating the square roots first. Show the steps that lead to your final answers.

   (a) \( \sqrt{9} + \sqrt{25} - \sqrt{64} \)  
   (b) \( 5\sqrt{4} + 2\sqrt{81} \)

   (c) \( \frac{2\sqrt{25} + 2}{3} \)  
   (d) \( \frac{1}{\sqrt{4}} \left( \sqrt{121} - \sqrt{9} \right) \)

All of the square roots so far have been “nice.” We will discuss what this means more in the next lesson. We can use the Multiplication Property to help simplify certain products of not-so-nice square roots.

3. Find each of the following products by first multiplying the radicands (the numbers under the square roots).

   (a) \( \sqrt{2} \cdot \sqrt{50} = \)  
   (b) \( \sqrt{3} \cdot \sqrt{12} = \)  
   (c) \( 5\sqrt{6} \cdot \sqrt{24} = \)

   (d) \( \sqrt{25} - \sqrt{2} \cdot \sqrt{8} = \)  
   (e) \( \frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{18}} = \)  
   (f) \( \frac{3}{\sqrt{4}} \cdot \frac{27}{\sqrt{4}} = \)
4. Write each of the following in simplest radical form. Show the work that leads to your answer. The first exercise has been done to remind you of the procedure.

(a) \( \sqrt{162} = \)

\[ = \sqrt{81} \cdot \sqrt{2} \]

\[ = 9\sqrt{2} \]

(b) \( \sqrt{20} = \)

(c) \( -\sqrt{90} = \)

(d) \( \sqrt{48} = \)

(e) \( -\sqrt{8} = \)

(f) \( \sqrt{300} = \)

5. Write each of the following products in simplest radical form. The first is done as an example for you.

(a) \( 3\sqrt{12} = \)

\[ = 3 \cdot \sqrt{4} \cdot \sqrt{3} \]

\[ = 3 \cdot 2 \cdot \sqrt{3} \]

\[ = 6\sqrt{3} \]

(b) \( 4\sqrt{45} = \)

(c) \( \frac{1}{2} \sqrt{32} = \)

(d) \( -2\sqrt{288} = \)

(e) \( \frac{\sqrt{108}}{3} = \)

(f) \( \frac{-\sqrt{320}}{16} = \)

**Reasoning**

It is critical to understand that when we “simplify” a square root or perform any calculation using them, we are always finding equivalent numerical expressions. Let’s make sure we see that in the final exercise.

6. Consider \( \sqrt{28} \).

(a) Use your calculator to determine its value. Round to the nearest hundredth.

(b) Write \( \sqrt{28} \) in simplest radical form.

(c) Use your calculator to find the value of the product from part (b). How does it compare to your answer from (a)?

(d) Do the same comparison for \( \sqrt{80} \).

Decimal Approximation: \( \sqrt{80} = \)

Simplified and then Approximated: \( \sqrt{80} = \)
The set of real numbers is made up of two distinctly different numbers. Those that are rational and those that are irrational. Their technical definitions are given below.

**RATIONAL AND IRRATIONAL NUMBERS**

1. A **rational number** is any number that can be written as the ratio of two integers. Such numbers include $\frac{3}{4}$, $\frac{-7}{3}$, and $\frac{5}{1}$. These numbers have terminating or repeating decimals.

2. An **irrational number** is any number that is not rational. So, ones that cannot be written as the ratio of two integers. These numbers have nonterminating and nonrepeating decimal representations.

**Exercise #1:** Let’s consider a number that is rational and one that is irrational (not rational). Consider the rational number $\frac{2}{3}$ and the irrational number $\sqrt{\frac{1}{2}}$. Both of these numbers are less than 1.

(a) Draw a pictorial representation of $\frac{2}{3}$ of the rectangle shown below.

(b) Using your calculator, give the decimal representation of the number $\frac{2}{3}$. Notice that it has a repeating decimal pattern.

(c) Write out all of the decimal places that your calculator gives you for $\sqrt{\frac{1}{2}}$. Notice that it does not have a repeating decimal pattern.

(d) Why could you not draw a pictorial representation of $\sqrt{\frac{1}{2}}$ that way you do for $\frac{2}{3}$?

Irrational numbers are necessary for a variety of reasons, but they are somewhat of a mystery. In essence they are a number that can never be found by subdividing an integer quantity into a whole number of parts and then taking an integer number of those parts. There are many, many types of irrational numbers, but square roots of non-perfect squares are always irrational. The proof of this is beyond the scope of this course.

**Exercise #2:** Write out every decimal your calculator gives you for these irrational numbers and notice that they never repeat.

(a) $\sqrt{2} =$

(b) $\sqrt{10} =$

(c) $\sqrt{23} =$
Rational and irrational numbers often mix, as when we simplify the square root of a non-perfect square.

**Exercise #3:** Consider the irrational number $\sqrt{28}$.

(a) Without using your calculator, between what two consecutive integers will this number lie? Why?

(b) Using your calculator, write out all decimals for $\sqrt{28}$.

(c) Write $\sqrt{28}$ in simplest radical form.

(d) Write out the decimal representation for your answer from (c). Notice it is the same as (b).

O.k. So, it appears that a non-zero rational number times an irrational number (see letter (c) above) results in an irrational number (see letter (d) above). We should also investigate what happens when we add rational numbers to irrational numbers (and subtract them).

**Exercise #4:** For each of the following addition or subtraction problems, a rational number has been added to an irrational number. Write out the decimal representation that your calculator gives you and classify the result as rational (if it has a repeating decimal) or irrational (if it doesn’t).

(a) $\frac{1}{2} + \sqrt{2}$

(b) $\frac{4}{3} + \sqrt{10}$

(c) $7 - \sqrt{8}$

**Exercise #5:** Fill in the following statement about the sum or rational and irrational numbers.

When a rational number is added to an irrational number the result is always ________________.

**Exercise #6:** Which of the following is an irrational number? If necessary, play around with your calculator to see if the decimal representation does not repeat. **Don’t be fooled by the square roots.**

(1) $\sqrt{25}$

(2) $4 - \sqrt{9}$

(3) $\frac{7}{2}$

(4) $3 + \sqrt{6}$
IRRATIONAL NUMBERS
COMMON CORE ALGEBRA I HOMEWORK

FLUENCY

1. For each of the following rational numbers, use your calculator to write out either the terminating decimal or the repeating decimal patterns.

(a) \( \frac{3}{4} \)  
(b) \( \frac{4}{9} \)  
(c) \( \frac{5}{8} \)  
(d) \( \frac{5}{6} \)

(e) \( \sqrt{\frac{25}{4}} \)  
(f) \( \sqrt{\frac{1}{100}} \)  
(g) \( \sqrt{\frac{4}{9}} \)  
(h) \( \sqrt{\frac{2}{32}} \)

2. One of the most famous irrational numbers is the number pi, \( \pi \), which is essential in calculating the circumference and area of a circle.

(a) Use your calculator to write out all of the decimals your calculator gives you for \( \pi \). Notice no repeating pattern.  
(b) Historically the rational number \( \frac{22}{7} \) has been used to approximate the value of \( \pi \). Use your calculator to write out all of the decimals for this rational number and compare it to (a).

3. For each of the following irrational numbers, do two things: (1) write the square root in simplest radical form and then (2) use your calculator to write out the decimal representation.

(a) \( \sqrt{32} \)  
(b) \( \sqrt{98} \)  
(c) \( \sqrt{75} \)  
(d) \( \sqrt{500} \)  
(e) \( \sqrt{80} \)  
(f) \( \sqrt{117} \)
**REASONING**

Types of numbers mix and match in various ways. The last exercise shows us a trend that we explored during the lesson.

4. Fill in the statement below based on the last exercise with one of the words below the blank.

The product of a (non-zero) rational number and an irrational number results in a(n) ____________ number.

Now we will explore other patterns in the following exercises.

5. Let’s explore the **product** of **two irrational numbers** to see if it is **always irrational, sometimes irrational, sometimes rational**, or **always rational**. Find each product below using your calculator (be careful as you put it in) and write out all decimals. Then, classify as either rational or irrational.

   (a) \( \sqrt{5} \cdot \sqrt{5} = \) ________________  
      Rational or irrational?

   (b) \( \sqrt{8} \cdot \sqrt{8} = \) ________________  
      Rational or irrational?

   (c) \( \sqrt{7} \cdot \sqrt{11} = \) ________________  
      Rational or irrational?

   (d) \( \sqrt{11} \cdot \sqrt{11} = \) ________________  
      Rational or irrational?

6. Based on #5, classify the following statement as true or false:

   **Statement**: The product of two irrational number is always irrational.  
   **True**  or  **False**

7. Let’s explore adding rational numbers. Using what you learned about in middle school, add each of the following pairs of rational numbers by first finding a **common denominator** then combine. Then, determine their repeating or terminating decimal.

   (a) \( \frac{1}{2} + \frac{2}{3} = \) 

   (b) \( \frac{3}{4} + \frac{1}{2} = \) 

   (c) \( \frac{3}{8} + \frac{5}{12} = \)

   (d) Classify the following statement as true or false:

   **Statement**: The sum of two rational numbers is always rational.  
   **True**  or  **False**

8. Finally, what happens when we add a rational and an irrational number (we explored this in Exercises #4 through #6 in the lesson). Fill in the blank below from what you learned in class.

   The sum of a rational number with an irrational number will always give a(n) ____________ number.

   **rational**  or  **irrational**
Square roots are operations on numbers that give exactly one output for a given input. So, they fit nicely into the definition of a function. We can graph the general square root function, once we establish a very important fact about square roots.

**Exercise #1:** Consider $\sqrt{-4}$?

(a) Why are neither 2 nor $-2$ the correct square root of $-4$?

(b) What can you conclude about taking square roots of negative numbers? Explain

It is absolutely critical that you understand, deep down inside, why finding the square root of a negative number is not possible with any real number. Let’s now get into the basic square root graph.

**Exercise #2:** Consider $f(x) = \sqrt{x}$.

(a) Create a table of values for input values of $x$ for which you can find rational square roots.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$f(x) = \sqrt{x}$</th>
</tr>
</thead>
</table>

(b) Graph the function on the grid provided.

(c) What is the domain of this function?

(d) What is the range of this function?

(e) Circle the correct choice below that characterizes $f(x) = \sqrt{x}$.

- $f(x)$ is always decreasing
- $f(x)$ is always increasing

(f) What shape does the square root graph appear to be “half” of? **This is not a coincidence.**
Square root graphs can be shifted just as quadratics can. And they shift in much the same way.

**Exercise #3:** The graph of \( y = \sqrt{x} \) is shown below.

(a) Using your calculator, graph the function given by \( y = \sqrt{x+4} + 2 \). Show your table of values.

(b) State the domain and range of this function.
   
   Domain: 
   
   Range: 

(c) Using your calculator, graph the function given by \( y = \sqrt{x-1} - 4 \). Show your table. Also state its domain and range.

<table>
<thead>
<tr>
<th>Table</th>
<th>Domain:</th>
<th>Range:</th>
</tr>
</thead>
</table>

So, it looks like our the shifting pattern that we saw with quadratics continues to hold with square root functions. This pattern would in fact hold no matter what function we were looking at. For example, let’s look back at our friend the absolute value function.

**Exercise #4:** The graph of \( y = |x| \) is shown on the grid below.

(a) Use your calculator to create a graph of \( y = |x+3| - 2 \).

(b) State the domain and range of this function:

   Domain: 
   
   Range: 

(c) Let’s see if you get the pattern. Sketch \( y = |x-2| - 1 \) without using your calculator.
**SQUARE ROOT FUNCTIONS AND SHIFTING**

**COMMON CORE ALGEBRA I HOMEWORK**

**FLUENCY**

1. Given the function \( f(x) = \sqrt{x-8} + 3 \), which of the following is the value of \( f(24) \)?
   - (1) 7
   - (2) 11
   - (3) 3
   - (4) 4

2. If \( g(x) = 4\sqrt{x} \) then \( g(45) \) is
   - (1) \( 7\sqrt{5} \)
   - (2) \( 12\sqrt{5} \)
   - (3) \( 36\sqrt{5} \)
   - (4) \( 22\sqrt{5} \)

3. Which of the following values of \( x \) is *not* in the domain of \( y = \sqrt{x-8} \)? Remember, the domain is the set of all inputs (x-values) that give an real output (y-value)?
   - (1) \( x = 12 \)
   - (2) \( x = 10 \)
   - (3) \( x = 8 \)
   - (4) \( x = 7 \)

4. Which of the following is the equation of the square root graph shown below?
   - (1) \( y = \sqrt{x+4} + 1 \)
   - (2) \( y = \sqrt{x+4} - 1 \)
   - (3) \( y = \sqrt{x-4} - 1 \)
   - (4) \( y = \sqrt{x-4} + 1 \)

5. Which of the following gives the range of the function \( y = |x-1| + 7 \)? Hint: Create a sketch by hand or on your calculator to help solve this problem.
   - (1) \( y \leq 1 \)
   - (2) \( y \geq 1 \)
   - (3) \( y \geq 7 \)
   - (4) \( y \leq 7 \)
APPLICATIONS

6. The bottom edge of a 16-foot long cantilever is given by the equation \( y = 2\sqrt{x} \), where \( y \) is the distance the bottom edge is from ground height, in feet.

(a) What is the value of \( b \), the height of the cantilever, in feet?

(b) To the nearest tenth of a foot, what is the thickness, \( T \), of the cantilever at \( x = 6 \) feet?

REASONING

7. On the grid shown to the right, \( y = \sqrt{x} \) is graphed. Without using your calculator, create a table and graph \( y = -\sqrt{x} \) on the same set of axes.

<table>
<thead>
<tr>
<th>( x )</th>
<th>0</th>
<th>1</th>
<th>4</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y = -\sqrt{x} )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Explain the effect on the graph of \( y = \sqrt{x} \) by multiplying by \(-1\).

8. Graph the function \( f(x) = -\sqrt{x+3} + 2 \) on the grid below. Show the table that you created by hand or using your calculator. Then, state its domain and range.

Table:

<table>
<thead>
<tr>
<th>( x )</th>
<th>0</th>
<th>1</th>
<th>4</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Domain: Range:
SOLVING QUADRATICS BY INVERSE OPERATIONS
COMMON CORE ALGEBRA I

Now that we have a good feeling for square roots, we can use them to help us solve special types of quadratic equations (those equations involving a squared quantity). Let’s make sure we first understand a basic concept.

**Exercise #1:** Solve each of the following equations for all values of $x$. Write your answers in simplest radical form.

(a) $x^2 = 16$  
(b) $x^2 = 100$  
(c) $x^2 = 20$

So, the key here is that the **inverse operation to squaring** is taking a **square root**. BUT, when you do this, you always introduce both a positive and negative answer. Squaring is a **non-reversible** process, meaning that you can’t simply undo it.

Now, let’s add some additional operations. Recall that we always solve equations by undoing operations in the opposite order in which they have been done. And in terms of order of operations, exponents essentially come first, so they will be “undone” last.

**Exercise #2:** Solve each of the following equations for all values of $x$ by using inverse operations. In each case your final answers will be rational numbers.

(a) $2x^2 + 10 = 28$  
(b) $\frac{x^2}{2} - 5 = 3$

(c) $(x - 2)^2 = 25$  
(d) $2(x + 5)^2 - 50 = 150$
Of course, there is no reason our answers must come out as rational numbers as in Exercise #3. We can also have answers to these types of equations that involve **irrational numbers**. In these cases we are typically asked, for some unknown reason, to express our answers in **simplest radical form**. 

**Exercise #3:** Solve each of the following quadratic equations by using inverse operations. Express all of your answers in simplest radical form.

(a) \(5x^2 - 2 = 38\)  
(b) \((x-3)^2 + 10 = 38\)

**Exercise #4:** Francis graphs the parabola \(y = \frac{1}{2}x^2 - 6\) on the grid below. He believes that the quadratic has zeroes of \(-3.5\) and \(3.5\). 

(a) Find the zeroes of this function in simplest radical form and explain why Francis must be incorrect.

(b) Francis was incorrect based on (a), but not too far off? How can you tell how good his estimate was?

**Exercise #5:** Find the zeroes of the function \(f(x) = (x+4)^2 - 20\) in simplest radical form. Then, express them in terms of a decimal rounded to the nearest **hundredth**.
SOLVING QUADRATICS USING INVERSE OPERATIONS
COMMON CORE ALGEBRA I HOMEWORK

1. Solve each of the following quadratics by applying inverse operations. In each case, your answers will be rational numbers. Always write them in simplest form.

(a) \( 2x^2 = 98 \)

(b) \( (x + 3)^2 = 25 \)

(c) \( x^2 - 11 = 53 \)

(d) \( \frac{(x - 4)^2}{3} = 12 \)

(e) \( 20(x + 1)^2 = 5 \)

(f) \( -2(x - 7)^2 + 5 = -195 \)

(g) \( (2x + 1)^2 - 6 = 19 \)

2. Which of the following is the solution set to the equation \( \frac{(x - 6)^2}{2} + 4 = 36 \) ?

(1) \( \{0, 12\} \)

(2) \( \{-4, 8\} \)

(3) \( \{-2, 16\} \)

(4) \( \{-2, 14\} \)
3. Solve each of the following quadratic equations by using inverse operations. Express each of your answers simplest radical form.

(a) \( \frac{1}{2}x^2 - 4 = 0 \)  \hspace{1cm} (b) \( (x - 5)^2 = 18 \)  \hspace{1cm} (c) \( 2x^2 + 7 = 71 \)

(d) \( 5(x + 2)^2 + 37 = 487 \)  \hspace{1cm} (e) \( \frac{(x - 4)^2}{3} + 8 = 17 \)

APPLICATIONS

4. The height, \( h \), of an object above the ground in feet, can be modeled as a function of time, \( t \), in seconds using the equation:

\[ h(t) = -16(t - 2)^2 + 400, \text{ for } t \geq 0 \]

(a) Find the time, in seconds, when the object reaches the ground, \( h = 0 \).

(b) Find all time(s) when the object is at a height of 200 feet. Round your answer(s) to the nearest tenth of a second.

5. Which of the following choices represent the zeroes of the function \( g(x) = 2(x - 5)^2 - 400 \)?

(1) \( x = 5 \pm 10\sqrt{2} \)  \hspace{1cm} (3) \( x = -5 \pm 20\sqrt{5} \)

(2) \( x = -10 \pm 20\sqrt{5} \)  \hspace{1cm} (4) \( x = 5 \pm 2\sqrt{10} \)
Finding Zeroes by Completing the Square
Common Core Algebra I

In the last lesson, we saw how to find the zeroes of a quadratic function if it was in vertex or shifted form. Do a warm-up problem to refresh this equation solving technique.

**Exercise #1:** For the quadratic function \( y = 2(x-2)^2 - 36 \).

(a) Find the zeroes in simplest radical form.  
(b) Find the zeroes to the nearest tenth.

But, of course, in order for us to find the zeroes using inverse operations as in (a), we need our quadratic in the form \( y = a(x-h)^2 + k \). In order to do this, we will use our technique of Completing the Square.

**Exercise #2:** Consider the quadratic \( y = x^2 - 6x - 16 \).

(a) Find the zeroes of this function by factoring.  
(b) Find the zeroes of this function by Completing the Square.

Now, it would probably seem to many students a bit redundant to know two methods for finding the zeroes of a quadratic function. Let’s illustrate why the technique of Completing the Square is important in its own right.

**Exercise #3:** Let’s take a look at the quadratic function \( y = x^2 + 6x + 2 \).

(a) Find the zeroes of this function using the method of Completing the Square. What kind of numbers are the solutions?  
(b) Try to factor \( x^2 + 6x + 2 \). Show your guesses and checks.

(c) What can you conclude about zeroes that are found using the Zero Product Law (Factoring)?
We now have a variety of tools at our disposal to find the **zeroes** and the **turning points** of quadratic functions. In one case we have the factored form of a quadratic; in a second case we have the vertex form of a quadratic. Each has its advantages and disadvantages.

**Exercise #4:** Let’s analyze the quadratic \( f(x) = 2x^2 - 4x - 16 \), which is written in **standard form**.

(a) Write the function in vertex form and state the coordinates of its turning point.  
(b) Using your answer from (a), find the zeroes of the function.

(c) Determine the function’s **y**-intercept.

Let’s see if we can now go in the opposite direction.

**Exercise #5:** The quadratic function pictured has a leading coefficient equal to 1. Answer the following questions based on your previous work.

(a) Write the equation of this quadratic in vertex form.

(b) Write the equation of this quadratic in factored form.

(c) How could you establish that these were **equivalent functions**?
FINDING ZEROES BY COMPLETING THE SQUARE
COMMON CORE ALGEBRA I HOMEWORK

FLUENCY

1. Solve the equation \( x^2 - 4x - 12 = 0 \) two ways:
   (a) By Factoring
   (b) By Completing the Square

2. Solve the equation \( x^2 + 10x + 21 = 0 \) two ways:
   (a) By Factoring
   (b) By Completing the Square

3. Find the solutions to the following equation in simplest radical form by using the technique of Completing the Square.
   \[ x^2 + 8x - 2 = 0 \]

4. Using the Method of Completing the square, find the zeroes of the following function to the nearest hundredth.
   \[ f(x) = 2x^2 + 12x + 5 \]
5. Consider the quadratic function shown below whose leading coefficient is equal to 1.

(a) Write the equation of this quadratic in $y = (x - h)^2 + k$ form.

(b) Find the zeroes of this quadratic in simplest radical form.

(c) Write the equation of this quadratic function in $y = ax^2 + bx + c$, i.e. standard, form.

6. Consider the quadratic function $y = x^2 + 2x - 48$ written in standard form.

(a) Write the quadratic function in its vertex form and state the coordinates of its turning point.

(b) Find the zeroes of the function algebraically by setting your equation from (a) equal to zero.

(c) State the range of this quadratic function. Justify your answer by creating a sketch of the function from what you found in (a) and (b).

(d) This quadratic can also be written in equivalent factored form as $y = (x - 6)(x + 8)$. What graphical features are easy to determine when the function is written in this form?

**REASONING**

7. Find the zeroes of the function $y = x^2 - 4x - 2$ in simplest radical form. Based on this answer, how do you know that you could not use factoring to find these zeroes?
THE QUADRATIC FORMULA
COMMON CORE ALGEBRA I

Our final topic in this unit looks at one of the most famous formulas in mathematics, the **Quadratic Formula**. The quadratic formula stems directly from the method of **Completing the Square**. Its proof or derivation is beyond the scope of this course. First, though, we begin with a Completing the Square Problem.

**Exercise #1:** Solve the equation \( x^2 + 8x + 3 = 0 \) by Completing the Square. What type of numbers do your answers represent?

Because of how algorithmic this process is, it can be placed in a formula:

For the quadratic equation \( ax^2 + bx + c = 0 \), the zeroes can be found by \( x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \).

**Exercise #2:** For the previous quadratic \( x^2 + 8x + 3 = 0 \) identify the following.

(a) The values of \( a \), \( b \), and \( c \) in the quadratic formula.

(b) Carefully substitute these values in the quadratic formula and simplify your expression. Compare your result to Exercise #1.

Students often prefer the Quadratic Formula to either **factoring** or **Completing the Square** to find the zeroes of a quadratic because it is so algorithmic in nature. Let’s compare it to factoring.

**Exercise #3:** Consider the quadratic equation \( 2x^2 - 9x + 4 = 0 \).

(a) Find the solutions to this equation by factoring.

(b) Find the solutions to this equation using the Quadratic Formula.
The Quadratic Formula is particularly nice when the solutions are **irrational numbers** and thus cannot be found by factoring. Sometimes, we have to place the answers to these equations in **simplest radical form** and sometimes we just need decimal approximations.

**Exercise #4:** For each of the following quadratic equations, find the solutions using the Quadratic Formula and express your answers in **simplest radical form**.

(a) \( x^2 + 6x - 9 = 0 \)  
(b) \( 3x^2 + 4x - 1 = 0 \)

Many times in applied problems it makes much greater sense to express the answers, even if irrational, as approximated decimals.

**Exercise #5:** A projectile is fired vertically from the top of a 60 foot tall building. It’s height in feet above the ground after \( t \)-seconds is given by the formula

\[
h = -16t^2 + 20t + 60
\]

Using your calculator, sketch a graph of the projectile’s height, \( h \), using the indicated window. At what time, \( t \), does the ball hit the ground? Solve by using the quadratic formula to the nearest **tenth** of a second.
THE QUADRATIC FORMULA

COMMON CORE ALGEBRA I HOMEWORK

FLUENCY

1. Solve the equation \( x^2 - 4x - 12 = 0 \) two ways:
   (a) By Factoring
   (b) By the Quadratic Formula

2. Solve the equation \( x^2 + 6x + 3 = 0 \) two ways. Express your answers both times in simplest radical form.
   (a) By Completing the Square
   (b) By the Quadratic Formula

3. Solve the equation \( 2x^2 - 13x + 20 = 0 \) two ways:
   (a) By Factoring
   (b) By the Quadratic Formula

4. If the quadratic formula is used to solve the equation \( x^2 - 4x - 41 = 0 \), the correct roots are
   (1) \( 4 \pm 3\sqrt{10} \)
   (2) \( 2 \pm 3\sqrt{5} \)
   (3) \( -4 \pm 3\sqrt{10} \)
   (4) \( -2 \pm 3\sqrt{5} \)
5. The quadratic function \( f(x) = x^2 - 12x + 31 \) is shown below.

(a) Find the zeroes of this function in simplest radical form by using the quadratic equation.

(b) Write this function in vertex form by completing the square. Based on this, what are the coordinates of its turning point? Verify on the graph.

APPLICATIONS

6. The flow of oil in a pipe, in gallons per hour, can be modeled using the function \( F(t) = -2t^2 + 20t + 11 \)

(a) Using your calculator, graph the function on the axes provided.

(b) Using the quadratic formula, find, to the nearest tenth of an hour, the time when the flow stops (is zero). Show your work.

(c) Use the process of completing the square to write \( F(t) \) in its vertex form. Then, identify the peak flow and the time at which it happens.
You now have a large variety of ways to solve quadratic equations, i.e. polynomial equations whose highest powered term is $x^2$. These techniques include factoring, Completing the Square, and the Quadratic Formula. In each application, it is essential that the equation that we are solving is equal to zero. If it isn’t, then some minor manipulation might be needed.

**Exercise #1:** Solve each of the following quadratic equations using the required method. First, arrange the equations so that they are set equal to zero.

(a) Solve by factoring:

$$x^2 + 5x - 12 = 8x - 2$$

(b) Solve by Completing the Square.

$$x^2 - 15x + 24 = -3x + 4$$

(c) Solve using the Quadratic Formula

Express answers to the nearest tenth.

$$x^2 - 3x + 16 = 5x + 15$$

(d) Solve using the Quadratic Formula

Express answers in simplest radical form.

$$x^2 + 4x + 2 = -2x + 7$$
Our final look at quadratic equations comes as a tie between their zeroes (where the functions cross the $x$-axis) and the algebraic solutions to find them.

**Exercise #2:** The quadratic $f(x) = (x - h)^2 + k$ is shown graphed on the grid below.

(a) What are the values of $h$ and $k$?

(b) What happens when you try to solve for the zeroes of $f$ given the values of $h$ and $k$ from part (a)? Why can’t you find solutions?

(c) How does what you found in part (b) show up in the graph to the right?

If you think about the graphs of parabolas, they can certainly “miss” the $x$-axis. When this happens **graphically** then when we solve for the zeroes **algebraically** we won’t be able to find any **real solutions** (although perhaps we will find some **imaginary ones** in Algebra II).

**Exercise #3:** Which of the following three quadratic functions has no real zeroes (there may be more than one). Determine by using the Quadratic Formula. Verify each answer by graphing in the standard viewing window.

\[
y = x^2 + 7x + 1 \quad \quad y = 3x^2 + 2x + 4 \quad \quad y = 5x^2 + 2x - 3
\]
COMMON CORE ALGEBRA I, UNIT #9 – ROOTS AND IRRATIONAL NUMBERS – LESSON #7

FLUENCY

1. Solve each of the following equations using the method described. Place your final answers in the form asked for.

(a) Solve by factoring: (Answers are exact)
   \[ 2x^2 - 2x + 1 = 4x + 1 \]

(b) Solve by factoring: (Answers are exact)
   \[ 2x^2 + 5x + 3 = x^2 + 9x + 15 \]

(c) Solve by Completing the Square (Round answers to the nearest tenth)
   \[ x^2 + 10x + 2 = 2x + 5 \]

(d) Solve using the Quadratic Formula (Express answers in simplest radical form)
   \[ 2x^2 + 3x - 3 = -3x - 4 \]

2. Which of the following represents the zeroes of the function \( f(x) = x^2 - 4x + 2 \)?

   (1) \( \{-1, 2\} \)  
   (2) \( \{2 - 2\sqrt{2}, 2 + 2\sqrt{2}\} \)  
   (3) \( \{2 - \sqrt{2}, 2 + \sqrt{2}\} \)  
   (4) \( \{-1, 4\} \)
APPLICATIONS

3. The percent of popcorn kernels that will pop, $P$, is modeled using the equation:

$$P = -0.03T^2 + 25T - 3600,$$
where $T$ is the temperature in degrees Fahrenheit.

Determine the two temperatures, to the nearest degree Fahrenheit, that result in zero percent of the kernels popping. Use the Quadratic Formula. Show work that justifies your answer. The numbers here will be messy. Use your calculator to help you and carefully write out your work.

REASONING

4. Find the zeroes of the function $y = x^2 - 4x - 16$ by Completing the square. Express your answers in simplest radical form. Graph the parabola using a standard window to see the irrational zeroes.

5. Explain how you can tell that the quadratic function $y = x^2 + 6x + 15$ has no real zeroes without graphing the function.

6. Use the Quadratic Formula to determine which of the two functions below would have real zeroes and which would not, then verify by graphing on your calculator using the STANDARD VIEWING WINDOW.

$$y = 2x^2 + 3x - 1$$

$$y = x^2 + 2x + 3$$
CUBE ROOTS
COMMON CORE ALGEBRA I

Just like square roots undo the squaring process, cube roots, undo the process of cubing a number. The cube root’s technical definition along with its symbolism is given below.

CUBE ROOTS

If \( x^3 = a \) then \( \sqrt[3]{a} \) is a solution to this equation. Or… \( \sqrt[3]{a} \) is any number that when cubed gives \( a \).

Exercise #1: It is good to know some basic cube roots of smaller numbers. Find each of the following and justify by using a multiplication statement.

(a) \( \sqrt[3]{8} \)  
(b) \( \sqrt[3]{1} \)  
(c) \( \sqrt[3]{27} \)

(d) \( \sqrt[3]{0} \)  
(e) \( \sqrt[3]{-1} \)  
(f) \( \sqrt[3]{-8} \)

One of the most striking differences between square roots and cube roots is that you can find the cube root of negative real numbers. For square roots, that will have to wait until you learn about non-real numbers in Algebra II.

Exercise #2: Using your calculator, use a guess and check scheme to find the following cube roots. Justify using a multiplication statement.

(a) \( \sqrt[3]{343} \)  
(b) \( \sqrt[3]{-2744} \)  
(c) \( \sqrt[3]{12,167} \)

Most calculators have a cube root option, although it may be harder to find than the square root button.

Exercise #3: Find each of the following cube roots to the nearest tenth by using your calculator’s cube root option/button.

(a) \( \sqrt[3]{100} \)  
(b) \( \sqrt[3]{-364} \)  
(c) \( \sqrt[3]{982} \)
The cube root also gives rise to the **cube root function**. Like the square root function, its basic graph is relatively easy to construct.

**Exercise #4:** Consider the basic cubic function \( y = \sqrt[3]{x} \).

(a) Fill out the table of values below without the use of your calculator.

<table>
<thead>
<tr>
<th>( x )</th>
<th>-8</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(b) Plot its graph on the grid provided below.

(c) Using your calculator, produce a graph to verify what you found in part (b).

Just like with all other functions, cube root graphs can be **transformed** in a variety of ways. Let’s see if our **shifting pattern** continues to hold with cube roots.

**Exercise #5:** Consider the function \( f(x) = \sqrt[3]{x+2} - 4 \).

(a) Use your calculator to create a table of values that can be plotted. Show your table below.

<table>
<thead>
<tr>
<th>( x )</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(b) Create a graph of this function on the axes provided.

(c) Describe how the graph you drew in Exercise #4 was shifted to produce this graph?
CUBE ROOTS
COMMON CORE ALGEBRA I HOMEWORK

FLUENCY

1. Find each of the following cube roots without the use of your calculator. Justify your answer based on a multiplication statement.
   (a) \( \sqrt[3]{8} \)  
   (b) \( \sqrt[3]{-1} \)  
   (c) \( \sqrt[3]{125} \)  
   (d) \( \sqrt[3]{0} \)  
   (e) \( \sqrt[3]{-8} \)  
   (f) \( \sqrt[3]{27} \)  
   (g) \( \sqrt[3]{\frac{1}{64}} \)  
   (h) \( \sqrt[3]{\frac{1}{1000}} \)

2. Use your calculator to find the following cube roots by trial and error. Justify your answers using a multiplication statement.
   (a) \( \sqrt[3]{512} \)  
   (b) \( \sqrt[3]{-2197} \)  
   (c) \( \sqrt[3]{9261} \)  
   (d) \( \sqrt[3]{-15,625} \)

3. The cube root function is the inverse of the cubing \( (x^3) \) function. Just as we can solve certain quadratic equations by using square roots, we can solve certain cubic equations by using cube roots. Solve each of the following in the form required. Use your calculator on (b) to find the cube root.
   (a) \( 2x^3 - 1 = 53 \) (Solve exactly)  
   (b) \( \frac{x^3}{8} - 3 = 7 \) (Solve to nearest tenth)

4. If \( g(x) = 5\sqrt[3]{x+7} - 4 \), then which of the following is the value of \( g(57) \)?
   (1) 19  
   (2) 11  
   (3) 16  
   (4) 25
5. Consider the function \( f(x) = \sqrt[3]{x} - 1 + 2 \) over the interval \(-7 \leq x \leq 9\).

(a) Graph \( f(x) \) over this domain interval only.

(b) State the range of the function over this interval.

(c) Recall that the average rate of change over the interval \( a \leq x \leq b \) is calculated by \( \frac{f(b) - f(a)}{b - a} \). Find the average rate of change of \( f(x) \) over the intervals below:

(i) \( 2 \leq x \leq 9 \)

(ii) \( 0 \leq x \leq 1 \)

(iii) \( -7 \leq x \leq 9 \)

6. The graph of \( y = \sqrt[3]{x} \) is shown below. On the same set of axes, graph \( f(x) = -2\sqrt[3]{x} \). Fill out the table below to help with your graph. What happened to the graph of \( y = \sqrt[3]{x} \) when multiplied by \(-2\)?)

<table>
<thead>
<tr>
<th>( x )</th>
<th>(-8)</th>
<th>(-1)</th>
<th>(0)</th>
<th>(1)</th>
<th>(8)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x) = -2\sqrt[3]{x} )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**REASONING**

6. Explain why it is not possible to find the square root of a negative number but it is possible to find the cube root of a negative number. Give examples to support your explanation.
UNIT #10

STATISTICS

Lesson #1 – Graphical Displays of Data
Lesson #2 – Quartiles and Box Plots
Lesson #3 – Measures of Central Tendency
Lesson #4 – Variation within a Data Set
Lesson #5 – Two Way Frequency Tables
Lesson #6 – Bivariate Data Analysis
Lesson #7 – Linear Regression on the Calculator
Lesson #8 – Other Types of Regression
Lesson #9 – Quantifying Predictability
Lesson #10 – Residuals
**GRAPHICALLY REPRESENTING DATA
COMMON CORE ALGEBRA I**

**Quantitative** data on a single variable is often collected in order to understand how a characteristic of a group differs amongst the group members or between groups. When we ask a question like “How old is a typical fast food worker?” it is helpful to take a survey and then see graphically how the ages differ amongst the group.

**Exercise #1:** Charlie's Food Factory currently employs 28 workers whose ages are shown below on a dot plot. Answer the following questions based on this plot.

(a) How many of the workers are 18 years old?  
(b) What is the range of the ages of the workers?

(c) Would you consider this distribution symmetric?  
(d) The mean (average) age for a worker is 22 years old. Why is this average not representative of a typical worker?

**Exercise #2:** A farm is studying the weight of baby chickens (chicks) after 1 week of growth. They find the weight, in ounces, of 20 chicks. The weights are shown below. Construct a dot plot on the axes given.

2, 1, 3, 4, 2, 2, 3, 1, 5, 3, 4, 4, 5, 6, 3, 8, 5, 4, 6, 3
**Exercise #3:** The following histogram shows the ages of the workers at Charlie’s Food Factory (from Exercise #1) but in a different format.

(a) How many workers have ages between 19 and 21 years?

(b) What is the disadvantage of a histogram compared to a dot plot?

(c) Does the histogram have any advantages over the dot plot?

**Exercise #4** The 2006 – 2007 Arlington High School Varsity Boy’s basketball team had an excellent season, compiling a record of 15 – 5 (15 wins and 5 losses). The total points scored by the team for each of the 20 games are listed below in the order in which the games were played:

76, 55, 76, 64, 46, 91, 65, 46, 45, 53, 56, 53, 57, 67, 58, 64, 67, 52, 58, 62

(a) Complete the frequency table below.

<table>
<thead>
<tr>
<th>POINTS SCORED</th>
<th>TALLY</th>
<th>FREQUENCY</th>
</tr>
</thead>
<tbody>
<tr>
<td>40 - 49</td>
<td></td>
<td></td>
</tr>
<tr>
<td>50 - 59</td>
<td></td>
<td></td>
</tr>
<tr>
<td>60 - 69</td>
<td></td>
<td></td>
</tr>
<tr>
<td>70 - 79</td>
<td></td>
<td></td>
</tr>
<tr>
<td>80 - 89</td>
<td></td>
<td></td>
</tr>
<tr>
<td>90 - 99</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(b) Construct the histogram below.
A local marketing company did a survey of 30 households to determine how many devices the household contained that family members watched video on (i.e. TV’s, tablets, smart phones, etcetera). The dot plot of the responses is shown below.

1. How many households have three devices capable of showing video on them?
   (1) 1  (3) 7
   (2) 2  (4) 5

2. More households had 4 devices to watch video on than any other number. Which of the following is closest to the percent of households that have 4 devices?
   (1) 22%  (3) 27%
   (2) 34%  (4) 45%

3. The marketing company would like to claim that the majority of households have either 3 or 4 screens capable of watching video on. Does the information displayed on the dot plot support this claim? Explain your reasoning.

4. The same marketing company then surveyed 30 households that contained at least one teenager. The dot plot for the video enabled devices is shown below. The mean number of screens for the first survey was 3.4. Based on the second dot plot, do you think its mean will be higher or lower? Explain.
On a recent Precalculus quiz, Mr. Weiler found the following distribution of scores, which are arranged in 5 point intervals (with the exception of the last interval).

5. How many students scored in the 75 to 79 point range?
   (1) 8  (3) 25
   (2) 10  (4) 5

6. Students do not pass the quiz if they receive lower than a 70. How many students did not pass?
   (1) 8  (3) 7
   (2) 5  (4) 15

7. How many total student took the quiz?
   (1) 25  (3) 56
   (2) 104  (4) 91

8. Twenty-two students scored in the 80 to 84 range on this test. Does the histogram provide us with enough information to conclude that a student must have scored on 82 on this test? Explain your thinking.

9. A random survey of 100 cars found the following frequency distribution for the fuel efficiency of the car, as measured in miles per gallon. Construct a histogram below that effectively shows the distribution of this data set.

<table>
<thead>
<tr>
<th>Fuel Efficiency (miles per gallon)</th>
<th>Number of Cars</th>
</tr>
</thead>
<tbody>
<tr>
<td>10 to 14</td>
<td>4</td>
</tr>
<tr>
<td>15 to 19</td>
<td>17</td>
</tr>
<tr>
<td>20 to 24</td>
<td>36</td>
</tr>
<tr>
<td>25 to 29</td>
<td>24</td>
</tr>
<tr>
<td>30 to 34</td>
<td>10</td>
</tr>
<tr>
<td>35 to 39</td>
<td>6</td>
</tr>
<tr>
<td>40 to 44</td>
<td>3</td>
</tr>
</tbody>
</table>
Another visual representation of how a data set is distributed comes in the form of a box plot. We create box plots by dividing the data up roughly into quarters by finding the quartiles of the data set.

**Exercise #1:** Shown below are the scores 16 students received on a math quiz.

52, 60, 66, 66, 68, 72, 72, 73, 74, 75, 80, 82, 84, 91, 92, 98

(a) What is the median of this data set?  
(b) Find the range of the data set (defined as the difference between the largest data value and the smallest data value).

(c) What is the median of the lower half of this data set (known as the **first quartile**, \( Q_1 \))?  
(d) What is the median of the upper half of this data set (known as the **third quartile**, \( Q_3 \))?  

The first and third quartiles are sometimes known as the lower and upper quartiles, respectively. The quartiles, the median, and the lowest and highest values in a data set comprise what is known as the **five number summary** and can be graphically represented on a box plot. This type of plot is also sometimes known as a box and whiskers plot.

**Exercise #3:** Using the same data set construct a box plot on the number line given below.
**Exercise #4:** The ages of the 15 employees of the Red Hook Curry House are given below.

16, 17, 17, 18, 19, 22, 25, 26, 29, 33, 33, 37, 40, 42, 44

(a) Determine the median and quartile values for this data set.

(b) Create a box-and-whiskers diagram below.

**Exercise #5:** Twenty of Mr. Ouimet’s physics students recently took a quiz. The results of this quiz are shown in the following box-and-whiskers diagram. Assume that all scores are whole numbers.

(a) What was the median score on Mr. Ouimet math quiz? 
(b) What was the range of the scores on Mr. Ouimet’s physics quiz?

(c) What score was greater than or equal to 75% of all other scores on this quiz? 
(d) Mr. Ouimet regularly sets the passing grade on his quizzes to be the score of the lower quartile. What is the passing grade on this quiz?

**Exercise #6:** Which of the following box plots shows a data set with the greatest median?

(1) 
(2) 
(3) 
(4)
QUARTILES AND BOX PLOTS
COMMON CORE ALGEBRA I HOMEWORK

FLUENCY

1. Which of the following data sets, given in ascending order, has the greatest range?

   (1) \{3, 4, 7, 10, 18\}  (3) \{-2, 5, 8, 11, 26\}
   (2) \{65, 66, 70, 72\}  (4) \{-5, -2, 4, 7, 10\}

2. Given the box plot shown below, which of the following represents the third quartile value for this data set?

   (1) 12  (3) 6
   (2) 18  (4) 19

3. Given the box plot shown below, which of the following represents the range of this data set?

   (1) 110  (3) 60
   (2) 40  (4) 75

4. According to the following box-and-whiskers diagram, which of the following values represents the lower quartile of this data set?

   (1) 20  (3) 28
   (2) 13  (4) 16

5. Which of the following box-and-whiskers diagram represents a data set whose median value is equal to 65?

   (1)  (3)
   (2)  (4)
APPLICATIONS

6. The ages of 12 retail workers are given in the data set below.

17, 18, 18, 19, 20, 21, 22, 23, 25, 25, 34, 47

(a) Calculate the five number summary. Label each of the five numbers with what they represent (i.e. min, max, lower quartile, etc.).

(b) Create a box-and-whiskers diagram of this data set below.

7. Mr. Ramirez gives a math test and records the grades of his 17 students as follows:

67, 72, 74, 74, 78, 80, 80, 82, 85, 85, 86, 87, 90, 92, 92, 95, 98

Create a box-and-whisker diagram of this data set below.

8. The speeds, in miles per hour, of 24 cars on a particular road are recorded and represented on the box-and-whiskers diagram shown below. Answer each of the following questions based on this diagram.

(a) What is the range of this data set?

(b) What is the maximum speed of the 24 drivers?

(c) How many drivers drove between 30 and 42 miles per hour?

(d) If the speed limit on this part of the road is 35 miles per hour, are more people speeding or are more people going below the speed limit? Justify.
MEASURES OF CENTRAL TENDENCY
COMMON CORE ALGEBRA I

In our day to day activities, we deal with many problems that involve related items of numerical information called data. Statistics is the study of sets of such numerical data. When we gather numerical data, besides displaying it, we often want to know a single number that is representative of the data as a whole. We call these types of numbers measures of central tendency. The two most common measures of central tendency are the mean and the median.

Exercise #1: A survey was taken amongst 12 people on the number of passwords they currently have to remember. The results in ascending order are shown below. State the median number of passwords and the mean number of passwords (to the nearest tenth).

0, 1, 1, 1, 2, 2, 3, 3, 3, 3, 4, 6

Exercise #2: Students in Mr. Okafor’s algebra class were trying to determine if people speed along a certain section of roadway. They collected speeds of 20 vehicles, as displayed in the table below.

(a) Find the mean and median for this data set.

<table>
<thead>
<tr>
<th>Speed (mph)</th>
<th>Number of Cars</th>
</tr>
</thead>
<tbody>
<tr>
<td>29</td>
<td>1</td>
</tr>
<tr>
<td>33</td>
<td>2</td>
</tr>
<tr>
<td>34</td>
<td>4</td>
</tr>
<tr>
<td>35</td>
<td>5</td>
</tr>
<tr>
<td>36</td>
<td>3</td>
</tr>
<tr>
<td>38</td>
<td>2</td>
</tr>
<tr>
<td>39</td>
<td>2</td>
</tr>
<tr>
<td>54</td>
<td>1</td>
</tr>
</tbody>
</table>

(b) The speed limit along this part of the highway is 35 mph. Based on your results from part (a), is it fair to make the conclusion that the average driver does speed on this roadway?
When conducting a statistical study, it is not always possible to obtain information about every person or situation to which the study applies. Unlike a census, in which every person is counted, some studies use only a sample or portion of the items being investigated. Whenever a sample is taken, it is vital that it be fair; in other words, the sample reflects the overall population.

**Exercise #3:** To determine which television programs are the most popular in a large city, a poll is conducted by selecting a sample of people at random and interviewing them. Outside which of the following locations would the interviewer be most likely to find a fair sample? Explain your choice and why the others are inappropriate.

1. A baseball stadium
2. A concert hall
3. A grocery store
4. A comedy club

**Exercise #4:** Truong is trying to determine the average height of high school male students. Because he is on the basketball team, he uses the heights of the 14 players on the team, which are given below in inches.

\[69, 70, 72, 72, 74, 74, 74, 75, 76, 76, 76, 77, 77, 82\]

(a) Calculate the mean and median for this data set. Round any non-integer answers to the nearest tenth.

(b) Is the data set above a fair sample to use to determine the average height of high school male students? Explain your answer.

Data sets can have members that are far away from all of the rest of the data set. These elements are called outliers, which can result in a mean that does not represent the true “average” of a data set.

**Exercise #5:** In Mr. Petrovic’s Advanced Calculus Course, eight students recently took a test. Their grades were as follows:

\[45, 78, 82, 85, 87, 89, 93, 95\]

(a) Calculate the mean and median of this data set.  
(b) What score is an outlier in this data set?

(c) Which value, the mean or the median, is a better measure of how well the average student did on Mr. Petrovic’s quiz?
MEASURES OF CENTRAL TENDENCY
COMMON CORE ALGEBRA I HOMEWORK

1. The Student Government at Arlington High School decided to conduct a survey to determine where to go on a senior field trip. They asked students the following question: “Would you rather go to a sports event or to an IMAX movie?” At which of the following locations would they most likely get a fair sample?

   (1) The gym, after a game  
   (2) The auditorium after a play  
   (3) A randomly chosen study hall  
   (4) At the Nature Club meeting.

2. For the following data set, calculate the mean and median. Any non-integer answers should be rounded to the nearest tenth.

   3, 5, 8, 8, 12, 16, 17, 20, 24

3. For the following data set, calculate the mean and median. Any non-integer answers should be rounded to the nearest tenth.

   5, 5, 9, 10, 13, 16, 18, 20, 22, 22

4. Which of the following is true about the data set \( \{3, 5, 5, 7, 9\} \)?

   (1) median > range  
   (2) median = mean  
   (3) mean > median  
   (4) median > mean

6. Which of the following data sets has a median of 7.5?

   (1) \( \{6, 7, 8, 9, 10\} \)  
   (2) \( \{3, 5, 7, 8, 10, 14\} \)  
   (3) \( \{1, 3, 7, 10, 14\} \)  
   (4) \( \{2, 7, 9, 11, 14, 17\} \)
7. A survey is taken by an insurance company to determine how many car accidents the average New York City resident has gotten into in the past 10 years. The company surveyed 20 people who are getting off a train at a subway station. The following table gives the results of the survey.

(a) Calculate the mean and median number of accidents of this data set. Remember, there are 6-zeros in this data set, 8-1’s, etc.

<table>
<thead>
<tr>
<th>Number of Accidents</th>
<th>Number of People</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>6</td>
</tr>
<tr>
<td>1</td>
<td>8</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>11</td>
<td>1</td>
</tr>
</tbody>
</table>

(b) Are there any outliers in this data set? If so, what data value?

(c) Which number, the mean or the median, better represents the number of accidents an average person in this survey had over this 10 year period? Explain your answer.

(d) Does this sample fairly represent the average number of accidents a typical New York City resident would get into over a 10 year period? Why or why not?

(e) Construct a dot plot that represents this data set on the axes below.

Is this a symmetric plot? Explain your thinking.
VARIATION WITHIN A DATA SET
COMMON CORE ALGEBRA I

Measures of central tendency give us numbers that describe the typical data value in a given data set. But, they do not let us know how much variation there is in the data set. Two data sets can have the same mean but look radically different depending on how varied the numbers are in the set.

Exercise #1: The two data sets below each have equal means but differ in the variation within the data set. Use your calculator to determine the Interquartile Range (IQR) of each data set. The IQR is defined as the difference between the third quartile value and the first quartile value.

Data Set #1: 3, 3, 4, 4, 5, 5, 6, 6, 7, 8, 8, 9, 9, 10, 10, 11, 11

Data Set #2: 5, 5, 6, 6, 7, 7, 8, 8, 9, 9

The interquartile range gives a good measure of how spread out the data set is. But, the best measure of variation within a data set is the standard deviation. The actual calculation of standard deviation is complex and we will not go into it here. We will rely on our calculators for its calculation.

Exercise #2: Using the same data sets above, use your calculator to produce the standard deviation (shown as $\sigma$ on the calculator) of the two data sets. Round your answers to the nearest tenth.

Data Set #1: 
Data Set #2: 

<table>
<thead>
<tr>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>The standard deviation of a data set tells us, on average, how far a data point is away from the mean of the data set. The larger the standard deviation, the greater the variation within the data set.</td>
</tr>
</tbody>
</table>

Exercise #3: A farm is studying the weight of baby chickens (chicks) after 1 week of growth. They find the weight, in ounces, of 20 chicks. The weights are shown below. Find the mean, the interquartile range and the standard deviation for this data set. Round any non-integer values to the nearest tenth. Include appropriate units in your answers. Give an interpretation of the standard deviation.

| 2, 1, 3, 4, 2, 2, 3, 1, 5, 3, 4, 4, 5, 6, 3, 8, 5, 4, 6, 3 |

mean interquartile range (IQR) standard deviation
Exercise #4: A marketing company is trying to determine how much diversity there is in the age of people who drink different soft drinks. They take a sample of people and ask them which soda they prefer. For the two sodas, the age of those people who preferred them is given below.

**Soda A:** 18, 16, 22, 16, 28, 18, 21, 38, 22, 29, 25, 44, 36, 27, 40

**Soda B:** 25, 22, 18, 30, 27, 19, 22, 28, 25, 19, 23, 29, 26, 18, 20

(a) Explain why standard deviation is a better measure of the diversity in age than the mean.

(b) Which soda appears to have a greater diversity in the age of people who prefer it? How did you decide on this?

(c) Use your calculator to determine the sample standard deviation, normally given as \( s \), for both data sets. Round your answers to the nearest tenth. Did this answer reinforce your pick from (b)? How?

Exercise #5: Which of the following data sets would have a standard deviation (population) closest to zero? Do this without your calculator. Explain how you arrived at your answer.

(1) \{-5, -2, -1, 0, 1, 2, 5\}  
(2) \{5, 8, 10, 16, 20\}  
(3) \{11, 11, 12, 13, 13\}  
(4) \{3, 7, 11, 11, 18\}
VARIATION WITHIN A DATA SET
COMMON CORE ALGEBRA I HOMEWORK

1. For each of the following data sets, use your calculator to help find the interquartile range and the population standard deviation. Show your calculation for the IQR. Round all non-integer values to the nearest tenth.
   (a) 4, 6, 8, 10, 15, 19, 22, 25
   (b) 3, 3, 4, 5, 5, 6, 6, 7, 7, 8

2. For the data set shown in the dot plot below, which of the following is closest to its population standard deviation?
   (1) 2.7   (3) 3.3
   (2) 4.2   (4) 5.8

3. What is the interquartile range of the data set represented in the box plot shown below?
   (1) 24   (3) 8
   (2) 14   (4) 12

4. Which of the following best measures the average distance that a data value lies away from the mean?
   (1) mean   (3) median
   (2) standard deviation   (4) range

5. Which of the following data sets would have the largest standard deviation?
   (1) \{3, 3, 4, 5, 5\}   (3) \{2, 8, 18, 26, 35\}
   (2) \{72, 73, 74, 75, 76\}   (4) \{8, 10, 12, 14, 16\}
6. We are going to revisit our survey of households that have video enabled devices (televisions, smart phones, tablets, etcetera). Recall that two surveys were done, each with 30 participants. In the first case (Survey A), the survey was random, in the second case (Survey B), the survey only included families with at least one teenager. The dot plots of the results are shown below.

![Survey A Dot Plot](image1)

![Survey B Dot Plot](image2)

(a) Enter the data into your calculator and use it to calculate the mean number of devices, the interquartile range, and the standard deviation of both data sets. Round all non-integers to the nearest tenth. Remember, you will have to enter a given data point more than once. For example, in Survey A, you will need to enter 2-0’s, 5-1’s, 3-2’s, etcetera. Use the sample standard deviation.

**Survey A Statistics:**

**Survey B Statistics:**

(b) Which of these two survey data sets had the greatest variation in the data? Explain based on the statistics you found in part (a).

(c) How many of the 30 values in Survey B fall within one standard deviation of the mean? To do this calculation, add the standard deviation and subtract the standard deviation from the mean and then count the number of values between the results of this addition and subtraction.
So far we have worked with **quantitative data** for a **single variable**, for example weight of baby chicks or number of video enabled devices. We can also work with **categorical data** or data that shows how many things **surveyed** fall into a **given category**.

**Exercise #1:** Let’s do a quick categorical survey in this class. By a show of hands, determine how many students fall into each of the following categories for eye color.

Brown Eyes  Blue Eyes  Green Eyes  Other

Although surveys of data that contain only one category are interesting, statisticians are often interested in how responses to two categories relate to one another. For example, we may want to know how a person’s gender (one category) affects what profession (a second category) they would prefer when they grow up. We may want to know if a person’s hair color (one category) has any relationship to their eye color (a second category). This type of data is summarized in a **two-way frequency table**.

**Exercise #2:** A class of 20 students recorded their hair color and eye color which are shown in the **two-way frequency table** below.

<table>
<thead>
<tr>
<th>Hair Color</th>
<th>Black</th>
<th>Blond</th>
<th>Red</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Blue</td>
<td>3</td>
<td>4</td>
<td>1</td>
<td>8</td>
</tr>
<tr>
<td>Brown</td>
<td>5</td>
<td>2</td>
<td>0</td>
<td>7</td>
</tr>
<tr>
<td>Green</td>
<td>1</td>
<td>1</td>
<td>3</td>
<td>5</td>
</tr>
<tr>
<td>Total</td>
<td>9</td>
<td>7</td>
<td>4</td>
<td>20</td>
</tr>
</tbody>
</table>

(a) How many students had blond hair and blue eyes?  (b) How many students had red hair?

(c) Construct a table that shows the **joint relative frequencies** and the **marginal relative frequencies** for the data above.
We would like to understand **associations** or **trends** within the data set, i.e. would a response to one category tell us something about the response to the other category?

**Exercise #3:** Let’s see if there is a connection between eye color and hair color by using **conditional relative frequencies**.

<table>
<thead>
<tr>
<th>Eye Color</th>
<th>Hair Color</th>
<th></th>
<th></th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Black</td>
<td>Blond</td>
<td>Red</td>
<td></td>
</tr>
<tr>
<td>Blue</td>
<td>3</td>
<td>4</td>
<td>1</td>
<td>8</td>
</tr>
<tr>
<td>Brown</td>
<td>5</td>
<td>2</td>
<td>0</td>
<td>7</td>
</tr>
<tr>
<td>Green</td>
<td>1</td>
<td>1</td>
<td>3</td>
<td>5</td>
</tr>
<tr>
<td>Total</td>
<td>9</td>
<td>7</td>
<td>4</td>
<td>20</td>
</tr>
</tbody>
</table>

(a) What is the conditional relative frequency of having green eyes if you have red hair? (This is equivalent to asking what the percent of people with red hair have green eyes.)

(b) What is the conditional relative frequency of having green eyes if you have black hair?

(c) Does it appear that having green eyes has a dependency or at least an association with having red hair? Explain.

(d) Is it more likely that a person with black hair has blue eyes or that a person with blond hair has brown eyes? Use conditional marginal frequencies to support your answer.

**Exercise #4:** A survey of 52 graduating seniors was conducted to determine if there was a connection between the gender of the student and whether they were going on to college. Based on this data, what is more likely: that someone going to college is female or that someone who is female is going to college? These may seem like the same thing, but are quite different.

<table>
<thead>
<tr>
<th>Gender</th>
<th>Male</th>
<th>Female</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Going to College</td>
<td>16</td>
<td>13</td>
<td>29</td>
</tr>
<tr>
<td>Not Going to College</td>
<td>14</td>
<td>9</td>
<td>23</td>
</tr>
<tr>
<td>Total</td>
<td>30</td>
<td>22</td>
<td>52</td>
</tr>
</tbody>
</table>
A survey was done to determine the relationship between gender and subject preference. A total of 56 students were surveyed to determine if they liked math, English, social studies, or science as their favorite subject. The results were then broken down based on whether the respondent was male or female.

<table>
<thead>
<tr>
<th></th>
<th>Math</th>
<th>English</th>
<th>Social Studies</th>
<th>Science</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Female</td>
<td>8</td>
<td>6</td>
<td>11</td>
<td>5</td>
<td>30</td>
</tr>
<tr>
<td>Male</td>
<td>10</td>
<td>4</td>
<td>8</td>
<td>4</td>
<td>26</td>
</tr>
<tr>
<td>Total</td>
<td>18</td>
<td>10</td>
<td>19</td>
<td>9</td>
<td>56</td>
</tr>
</tbody>
</table>

1. Which of the following is closest to the joint relative frequency of being a male who likes social studies?
   (1) 0.42   (2) 0.14   (3) 0.31   (4) 0.56

2. Which of the following is the marginal relative frequency of liking math?
   (1) \( \frac{18}{36} \)   (2) \( \frac{8}{10} \)   (3) \( \frac{10}{18} \)   (4) \( \frac{18}{56} \)

3. What percent of female students liked English as their favorite subject?
   (1) 20%   (2) 16%   (3) 11%   (4) 60%

4. A person looking at this table concludes that it is more likely that a female student will like social studies than a male student will like math. Is this correct? Justify your answer.

5. Is it more likely that a person who likes social studies will be female or that a person who is female will like social studies? Justify.
Demographers are trying to understand the association between where a person lives and how they commute to work. They survey 100 people in three cities with the results shown below.

<table>
<thead>
<tr>
<th></th>
<th>Car</th>
<th>Train</th>
<th>Walk</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>New York</td>
<td>5</td>
<td>25</td>
<td>10</td>
<td>40</td>
</tr>
<tr>
<td>Los Angeles</td>
<td>18</td>
<td>12</td>
<td>5</td>
<td>35</td>
</tr>
<tr>
<td>Chicago</td>
<td>8</td>
<td>14</td>
<td>3</td>
<td>25</td>
</tr>
<tr>
<td>Total</td>
<td>31</td>
<td>51</td>
<td>18</td>
<td>100</td>
</tr>
</tbody>
</table>

6. Fill in the table below with the relative frequencies.

<table>
<thead>
<tr>
<th></th>
<th>Car</th>
<th>Train</th>
<th>Walk</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>New York</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Los Angeles</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Chicago</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

7. Given that a person rides a train to work, what is the conditional relative frequency that they live in New York?
   (1) 0.25  (3) 0.49
   (2) 0.63  (4) 0.82

8. If a person lives in Los Angeles, what is the conditional relative frequency that they drive a car?
   (1) 0.42  (3) 0.68
   (2) 0.16  (4) 0.51

9. Which of the following is the marginal frequency of walking to work?
   (1) 18%  (3) 25%
   (2) 60%  (4) 44%

10. Is a person more likely to ride a train if they live in New York or if they live in Chicago? Justify your answer.
**BIVARIATE DATA ANALYSIS**

**COMMON CORE ALGEBRA I**

Oftentimes, statistical studies are done where data is collected on two variables instead of one in order to establish whether there is a relationship between the two variables. This is called a bivariate data analysis.

**Exercise #1:** A survey was taken of 10 low and high temperatures, in Fahrenheit, in the month of April to try to establish a relationship between a day’s low temperature and high temperatures.

<table>
<thead>
<tr>
<th>Low Temperature, $x$</th>
<th>26</th>
<th>28</th>
<th>30</th>
<th>32</th>
<th>34</th>
<th>35</th>
<th>37</th>
<th>38</th>
<th>41</th>
<th>45</th>
</tr>
</thead>
<tbody>
<tr>
<td>High Temperature, $y$</td>
<td>49</td>
<td>50</td>
<td>57</td>
<td>54</td>
<td>60</td>
<td>58</td>
<td>64</td>
<td>66</td>
<td>63</td>
<td>72</td>
</tr>
</tbody>
</table>

(a) Construct a scatter plot of this bivariate data set on the grid below.

(b) Draw a line of best fit through this data set.

(c) Calculate the slope of this line by picking off two points (not necessarily data points).

(d) Use your line of best fit to estimate the high temperature for a day in April given that the low temperature was 42 degrees. Illustrate your answer on your graph.

(e) Would you characterize the relationship between the low and high temperature as a positive correlation or a negative correlation? Explain.
Two variables can have a strong relationship with one another, as seen on a scatterplot, but might not have a causal relationship. A causal relationship exists when the change in one variable actually causes the change in the other (or is one of the primary causes).

Exercise #2: In each of the following scenarios, two variables are given that if plotted would have a strong correlation (a scatterplot where the data falls nearly in a line). Determine if there exists a causal relationship between the two variables. If so, which variable causes the other?

(a) The high temperature in New York City and the number of bottles of water sold.
(b) A person’s height and a person’s shoe size.
(c) A person’s weight loss and the number of hours a person spends in the gym per week.
(d) The years of education a person achieves and the salary that person starts at upon entering the work force.

Variables can have extremely strong correlations but no causal relationship. This is often the case if there is a third variable that causes both (known interestingly enough as a lurking variable).

Exercise #3: The table below shows the number of firefighters required to fight a given fire versus the dollar damage done to the house by the fire.

<table>
<thead>
<tr>
<th>Number of firefighters</th>
<th>2</th>
<th>3</th>
<th>5</th>
<th>8</th>
<th>9</th>
<th>12</th>
<th>16</th>
</tr>
</thead>
<tbody>
<tr>
<td>Damage done by fire (in dollars)</td>
<td>2,932</td>
<td>9,750</td>
<td>15,575</td>
<td>23,190</td>
<td>22,900</td>
<td>35,400</td>
<td>52,900</td>
</tr>
</tbody>
</table>

(a) Are the data positively or negatively correlated? How can you tell?  
(b) Does the number of firefighters cause the damage done to the house? If not, what hidden variable is causing both variables to change?
BIVARIATE DATA ANALYSIS
COMMON CORE ALGEBRA I HOMEWORK

1. A survey was done at Ketcham High School to determine the effect of time spent on studying and grade point average. The table below shows the results for 10 students randomly selected.

<table>
<thead>
<tr>
<th>Study time (Hours per week)</th>
<th>2</th>
<th>4</th>
<th>5</th>
<th>7</th>
<th>10</th>
<th>12</th>
<th>14</th>
<th>17</th>
<th>19</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>GPA (out of 100)</td>
<td>64</td>
<td>71</td>
<td>69</td>
<td>74</td>
<td>81</td>
<td>86</td>
<td>84</td>
<td>94</td>
<td>91</td>
<td>96</td>
</tr>
</tbody>
</table>

(a) Create a scatter plot for this data set on the grid provided. Draw a best fit line through the scatterplot drawn.

(b) Determine the equation for the best fit line for this data set. Use two points from the line you drew in (a) to determine the slope and estimate the $y$-intercept graphically. Round your slope to the nearest tenth and the $y$-intercept to the nearest whole number.

(c) Use your answer from part (b) to determine the expected GPA from studying for 8 hours per week. Round your answer to the nearest whole number.

(d) Is there a causal relationship between these two variables? If so, which variable causes the other?
2. A survey was done to determine if there was any connection between the price that people pay for their most expensive car and the current value of their house. The results, for eight participants, are given below.

<table>
<thead>
<tr>
<th>Car Cost, $x$ (in dollars)</th>
<th>11,500</th>
<th>14,750</th>
<th>19,500</th>
<th>26,750</th>
<th>32,900</th>
<th>43,000</th>
<th>45,750</th>
<th>54,500</th>
</tr>
</thead>
<tbody>
<tr>
<td>House Value, $y$ (in dollars)</td>
<td>160,000</td>
<td>195,000</td>
<td>255,000</td>
<td>400,000</td>
<td>440,000</td>
<td>525,000</td>
<td>475,000</td>
<td>725,000</td>
</tr>
</tbody>
</table>

A computer was used to determine the line of best fit. Its equation was:

$$y = 12x + 33,766$$

(a) Use the line of best fit to predict the house value of a person whose most expensive car costs $19,500.

(b) Was the prediction in (a) an overestimate or underestimate of the actual house value? Explain.

(c) Is there a positive or negative correlation between these two variables? Explain.

(d) Is there a causal relationship between these two variables? If you answer yes, then determine which variable causes the other. If you answer no, then explain a third variable that could be causing both.

3. It has been noted that on any given day, there is a strong correlation between the number of ice-cream cones sold and the number of people who go swimming. Is there a causal relationship between the eating ice-cream and going swimming? If not, what could be causing this strong correlation?
LINEAR REGRESSION ON THE CALCULATOR
COMMON CORE ALGEBRA I

In the last lesson we drew lines of best fit by hand so that we could model bivariate data that had been graphed on a scatter plot. In this lesson, we will see how to use our calculators to find the equation of the line of best fit. This is often referred to as linear regression. We will work in the first exercise with a data set from the last lesson.

Exercise #1: A survey was taken of 10 low and high temperatures, in Fahrenheit, in the month of April to try to establish a relationship between a day’s low temperature and high temperatures.

<table>
<thead>
<tr>
<th>Low Temperature, ( x )</th>
<th>26</th>
<th>28</th>
<th>30</th>
<th>32</th>
<th>34</th>
<th>35</th>
<th>37</th>
<th>38</th>
<th>41</th>
<th>45</th>
</tr>
</thead>
<tbody>
<tr>
<td>High Temperature, ( y )</td>
<td>49</td>
<td>50</td>
<td>57</td>
<td>54</td>
<td>60</td>
<td>58</td>
<td>64</td>
<td>66</td>
<td>63</td>
<td>72</td>
</tr>
</tbody>
</table>

(a) Enter data into lists on your calculator. And create a scatter plot using your graphing technology. Size the WINDOW appropriately so that the data takes up the majority of the screen. Compare this to the scatter plot you created in the last lesson.

(b) Use your calculator to find the equation for the line of best fit. Round the slope of the line to the nearest hundredth and the \( y \)-intercept to the nearest integer. Compare this to the slope you found in the last lesson.

Equation for Best Fit Line  
Slope from Lesson #6

(c) Explain what the \( y \)-intercept of this model represents in terms of the low and high temperatures that are being modeled in this problem.

(d) How would you interpret the slope of this model in terms of how the low and high temperatures change with respect to each other?
**Exercise #2:** Generally, the fuel efficiency of a car changes with the weight of the car. A survey of some cars with their weights and gas mileages is shown below.

<table>
<thead>
<tr>
<th>Weight (1000’s of lbs)</th>
<th>3.7</th>
<th>4.5</th>
<th>3.2</th>
<th>5.1</th>
<th>6.8</th>
<th>4.9</th>
<th>4.8</th>
<th>5.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mileage (miles per gallon)</td>
<td>38</td>
<td>26</td>
<td>48</td>
<td>24</td>
<td>18</td>
<td>30</td>
<td>28</td>
<td>21</td>
</tr>
</tbody>
</table>

(a) Find the equation for the line of best fit using your calculator. Round both coefficients to the nearest tenth. List what the variables $x$ and $y$ represent in this problem.

(b) Create a graph of the scatter plot for this data. Would you consider the **correlation** between weight and mileage to be **positive** or **negative**? Explain.

(c) Which **parameter** of the linear model predicts whether the **correlation** is positive or negative? Use this model to help explain your answer.

(d) If a car had a weight of 4,300 pounds, what would this model predict as its fuel efficiency? Round to the nearest integer. Use appropriate units and make sense of your answer.

(e) If we wanted to purchase a car that got 40 miles to a gallon, what weight of car, to the nearest 100 pounds, should we purchase? Solve algebraically.
LINEAR REGRESSION ON THE CALCULATOR
COMMON CORE ALGEBRA I HOMEWORK

1. We are now going to revisit our data from the homework yesterday, but with our calculator. A survey was done at Ketcham High School to determine the effect of time spent on studying and grade point average. The table below shows the results for 10 students randomly selected.

<table>
<thead>
<tr>
<th>Study time (Hours per week)</th>
<th>2</th>
<th>4</th>
<th>5</th>
<th>7</th>
<th>10</th>
<th>12</th>
<th>14</th>
<th>17</th>
<th>19</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>GPA (out of 100)</td>
<td>64</td>
<td>71</td>
<td>69</td>
<td>74</td>
<td>81</td>
<td>86</td>
<td>84</td>
<td>94</td>
<td>91</td>
<td>96</td>
</tr>
</tbody>
</table>

(a) Enter the data in your calculator and use it to generate the equation for the line of best fit. Round your slope to the nearest tenth and round your $y$-intercept to the nearest integer.

(b) According to the linear regression model from part (a), what GPA, to the nearest integer, would result from studying for 15 hours in a given week? Justify your answer.

(c) A passing average is defined as a 65% or above. Does the model predict a passing average if the student spends no time studying in a given week? Justify your answer.

(d) For each additional hour that a student studies per week, how many points does the model predict a GPA will rise? Explain how you arrived at your answer.

(e) Create a scatter plot of this data on your calculator. State the WINDOW that you used below. Compare this scatter plot to the one that you created by hand on the previous homework.

WINDOW: $x_{\text{min}} =$, $x_{\text{max}} =$, $y_{\text{min}} =$, $y_{\text{max}} =$
2. The mean annual temperature of a location generally depends on its elevation above sea level. A collection of nine locations in Nevada were chosen and had their elevation and mean annual temperature recorded. The data is shown below.

<table>
<thead>
<tr>
<th>Elevation (feet)</th>
<th>1200</th>
<th>4125</th>
<th>6230</th>
<th>2378</th>
<th>5625</th>
<th>6328</th>
<th>4375</th>
<th>1864</th>
<th>3160</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean Temperature (°F)</td>
<td>62</td>
<td>45</td>
<td>36</td>
<td>51</td>
<td>48</td>
<td>32</td>
<td>40</td>
<td>58</td>
<td>49</td>
</tr>
</tbody>
</table>

(a) Use your calculator to determine the equation for the line of best fit. Round your slope to the nearest thousandth. Note that it will be a small number. Round your y-intercept to the nearest integer.

(b) What does the y-intercept tell you about the temperature in Nevada?

(c) Using correct units, give an interpretation of the slope of this line.

(d) Using your model from part (a), what would be the predicted mean temperature at an elevation of 3000 feet above sea level?

(e) Would you characterize this correlation as being positive or negative? How can you tell this from the equation itself?

(f) Create a scatter plot of the data and graph the line of best fit on it as well. Are there any data points from the table above that are significantly “missed” by the model? If so, which data point?
In the last two lessons we fit bivariate data sets with lines of best fit. Sometimes, though, linear models are not the best choice. We can fit data with all sorts of curves, the most common of which are linear, exponential, and quadratic. But, there are many other types. Before we look at exponential and quadratic regression, recall the general shapes of these two types of functions.

**Exercise #1:** For each scatterplot shown below, determine if it is best fit with a linear, exponential, or quadratic function. Draw a curve of best fit depending on your choice.

(a) [Diagram]  
Type: ________________

(b) [Diagram]  
Type: ________________

(c) [Diagram]  
Type: ________________

(d) [Diagram]  
Type: ________________

(e) [Diagram]  
Type: ________________

(f) [Diagram]  
Type: ________________
Our calculators can produce equations for **exponentials of best fit** and **quadratics of best fit** (along with a lot of other types of curves).

**Exercise #2:** Biologists are modeling the number of flu cases as it spreads around a particular city. The total number of cases, \( y \), was recorded each day, \( x \), after the total first reached 16. The data for the first week is shown in the table below.

<table>
<thead>
<tr>
<th>( x ), days</th>
<th>0</th>
<th>1</th>
<th>3</th>
<th>4</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y ), cases</td>
<td>16</td>
<td>18</td>
<td>22</td>
<td>25</td>
<td>33</td>
<td>35</td>
</tr>
</tbody>
</table>

(a) Use your calculator to find the **exponential regression equation** for this data set in the form 
\[ y = a(b)^x \]
Round all parameters to the nearest hundredth.

(b) Based on the regression equation, how many total cases of flu will there be after two weeks?

(c) According to your model, by what percent are the flu cases increasing on a daily basis?

(d) Hospital officials will declare an emergency when the total number of cases exceeds 200. On what day will they need to declare this emergency?

So, really, regression, as mysterious as it may be, is all about finding the best version of whatever curve we think fits the data best.

**Exercise #3:** The cost per widget produced by a factory generally drops as more are produced but then starts to rise again due to overtime costs and wear on the equipment. Quality control engineers recorded data on the cost per widget compared to the number of widgets produced. Their data is shown below.

<table>
<thead>
<tr>
<th>Number of widgets, ( x )</th>
<th>35</th>
<th>88</th>
<th>110</th>
<th>135</th>
<th>154</th>
<th>190</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost per widget, ( y )</td>
<td>9.32</td>
<td>2.63</td>
<td>1.42</td>
<td>1.32</td>
<td>2.12</td>
<td>5.50</td>
</tr>
</tbody>
</table>

(a) Why should a quadratic model be considered for this data set as opposed to linear or exponential?

(b) Use your calculator to create a scatterplot of this data to verify its quadratic nature.
OTHER TYPES OF REGRESSION
COMMON CORE ALGEBRA I HOMEWORK

FLUENCY

1. For each scatterplot below, determine the best type of regression from: linear, exponential, or quadratic. Draw a representative curve (line, exponential, or parabola) through the data.

(a) (b) (c)  
Type: ________________  Type:__________________  Type: _______________

(d)  (e) (f)  
Type: _______________  Type: ________________  Type: _______________

2. Given the scatterplot below, which of the following equations would best model the data? Explain your choice.

(1) \( y = -3x + 6 \)  
(2) \( y = 6(2)^x \)  
(3) \( y = -4x^2 + 20x + 3 \)  
(4) \( y = 2x^2 - 6x + 4 \)
APPLICATIONS

3. A marketing company is keeping track of the number of hits that a website receives on a daily basis. Their data for the first two weeks is shown below. A scatterplot of the data is also shown.

<table>
<thead>
<tr>
<th>Days</th>
<th>Hits</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>120</td>
</tr>
<tr>
<td>3</td>
<td>145</td>
</tr>
<tr>
<td>5</td>
<td>162</td>
</tr>
<tr>
<td>10</td>
<td>220</td>
</tr>
<tr>
<td>14</td>
<td>270</td>
</tr>
</tbody>
</table>

(a) Of the three types of regression we have studied which seems least likely to fit this data? Explain your choice.

(b) Find a linear equation, in the form \( y = ax + b \), that best models this data and an exponential equation, in the form \( y = a(b)^x \) that best models this data. Round all parameters to the nearest hundredth.

<table>
<thead>
<tr>
<th>Linear Model</th>
<th>Exponential Model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(c) How close are the two model’s outputs when \( x = 10 \)? Show the values you find.

(d) How close are the two model’s outputs when \( x = 30 \)? Show the values that you find.

(e) Which model will predict faster growth of website hits over time? Explain your answer. You may want to experiment by graphing both models.
QUANTIFYING PREDICTABILITY
COMMON CORE ALGEBRA I

In the last few lessons we have worked with generating lines and curves of best fit for bivariate data sets. In every circumstance, though, the data did not fall on a straight line or on a perfect curve. We have never answered the question of how well specifically a linear model does in predicting the correlation between the two variables.

Exercise #1: Your teacher will explain how to ensure that your calculator has its “r-value” on. Since this varies by graphing calculator, write down the procedure below if necessary.

Exercise #2: In the following exercises four data sets with equal x-values are given to illustrate different types of positive correlations. For each, enter the data, observe the scatter plot, and record the r-value, known as the correlation coefficient, for a linear fit to the nearest thousandth.

(a)  
\[
\begin{array}{c|cccccc}
  x & 2 & 5 & 8 & 11 & 15 & 18 \\
  y & 4 & 13 & 22 & 29 & 43 & 52 \\
\end{array}
\]

(b)  
\[
\begin{array}{c|cccccc}
  x & 2 & 5 & 8 & 11 & 15 & 18 \\
  y & 16 & 14 & 22 & 41 & 37 & 51 \\
\end{array}
\]

(c)  
\[
\begin{array}{c|cccccc}
  x & 2 & 5 & 8 & 11 & 15 & 18 \\
  y & 18 & 8 & 41 & 28 & 62 & 44 \\
\end{array}
\]

(d)  
\[
\begin{array}{c|cccccc}
  x & 2 & 5 & 8 & 11 & 15 & 18 \\
  y & 44 & 51 & 30 & 55 & 45 & 47 \\
\end{array}
\]

(d) How does the correlation coefficient quantify the fit of a positive correlation?

Exercise #3: The following data set is that of two variables that have a negative correlation. Enter the data, produce the scatter plot, and record the r-value. How is the negative correlation reflected in the r-value?

\[
\begin{array}{c|cccccc}
  x & 2 & 5 & 8 & 11 & 15 & 18 \\
  y & 52 & 47 & 28 & 32 & 25 & 10 \\
\end{array}
\]
Exercise #4: Given the scatter plot shown below, which of the \( r \)-values would most likely represent the correlation between the two variables? Explain your choice.

(1) \( r = 0.88 \) 
(2) \( r = 0.28 \) 
(3) \( r = 1 \) 
(4) \( r = -0.94 \)

Exercise #5: Which of the following scatter plots would have a correlation coefficient closest to \(-1\)?

Exercise #6: There are two primary types of crude oil sold in the world, West Texas Intermediate (WTI) and Brent Crude. Each is priced differently on a daily basis and each has a correlation with the average price per gallon for unleaded gasoline. The two linear regression models, along with their \( r \)-values, are shown below. Give a prediction for the price per gallon of unleaded gasoline, \( y \), on a day when the price for WTI is \$103 and the price for Brent is \$109, \( x \). Which model did you choose and why?

Brent Crude: \( y = 0.028x + 0.71, r = 0.973 \)  
WTI Crude: \( y = 0.031x + 0.67, r = 0.924 \)
QUANTIFYING PREDICTABILITY
COMMON CORE ALGEBRA I HOMEWORK

1. Below there are six scatter plots, six correlation coefficients, and six terms. Match the appropriate $r$-value with the scatter plot it most likely corresponds to. Then match the term you think is most appropriate to the $r$-value as well (not to the graph).

(a) $r = 1.0$  Weak Negative

(b) $r = 0.35$  Perfect Positive

(c) $r = -0.82$  Strong Positive

(d) $r = 0$  Weak Positive

(e) $r = -0.56$  Moderate Negative

(f) $r = 0.93$  No Correlation
2. A solar power company is trying to correlate the total possible hours of daylight (simply the time from sunrise to sunset) on a given day to the production from solar panels on a residential unit. They created a scatter plot for one such unit over the span of five months. The scatter plot is shown below.

The equation line of best fit for this bivariate data set was:

\[ y = 2.26x + 20.01 \].

(a) How many kilowatt hours would the model predict on a day that has 14 hours of possible daylight?

(b) To the nearest tenth of an hour, how many hours of possible daylight would be needed to produce 50 kilowatt hours of energy?

(c) The correlation coefficient for this regression was \( r = 0.134 \). Would you characterize this as strongly positive, moderately positive, or a weakly positive correlation? Explain.

(d) Based on (c), do you have confidence in the model to accurately predict the energy production based on the total possible daylight hours? Explain.

(e) What environmental factors might contribute to the “noise” in the data? Noise are factors that prevent the correlation from being perfect.
RESIDUALS
COMMON CORE ALGEBRA I

In the last lesson, we saw how the correlation coefficient (or r-value) measures the predictability of the model (or how well it will do in its predictions). Although the r-value is an excellent measure, it does not tell us whether the model is appropriate only whether it does a good job at predicting. Today we will examine what are known as residuals and residual graphs to determine if a linear model makes sense.

**Exercise #1:** A skydiver jumps from an airplane and an attached micro-computer records the time and speed of the diver for the first 12 seconds of the diver’s freefall. The data is shown in the table below.

<table>
<thead>
<tr>
<th>Time (sec)</th>
<th>0</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
<th>10</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>Speed (ft/sec)</td>
<td>0</td>
<td>25</td>
<td>46</td>
<td>60</td>
<td>68</td>
<td>72</td>
<td>74</td>
</tr>
</tbody>
</table>

(a) Find the equation for the line of best fit for this data set. Round both coefficients to the nearest tenth. As well, determine the correlation coefficient and round it to the nearest hundredth. Based on the correlation coefficient, characterize the fit as positive or negative and how strong of a fit it is. Create a plot with both the data and line of best fit shown on it. You do not need to reproduce the plot below.

(b) The residual of a data point is defined as the difference between the observed y-value and the predicted y-value. Using tables on your calculator fill in the table below with the predicted values (rounded to the nearest integer) and the residuals for each data point.

<table>
<thead>
<tr>
<th>Time (sec)</th>
<th>0</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
<th>10</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>Speed (ft/sec)</td>
<td>0</td>
<td>25</td>
<td>46</td>
<td>60</td>
<td>68</td>
<td>72</td>
<td>74</td>
</tr>
<tr>
<td>Prediction (ft/sec)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Residual (ft/sec)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(c) Sketch a plot of the residuals below. Your teacher will need to show you how to do this on your graphing calculator. Make sure all other scatter plots and equations are off. Do the residuals show any distinct pattern?
Generally, we do not want residuals to fall along a curve or make a distinct pattern. If so, then it is likely that a linear model is not appropriate to fit the data and perhaps an exponential or quadratic model is better.

**Exercise #2:** A school district was attempting to correlate the number of hours a student studies in a given week with their grade point average. They surveyed 8 students and found the following data.

<table>
<thead>
<tr>
<th>Hours Studying</th>
<th>3</th>
<th>7</th>
<th>2</th>
<th>11</th>
<th>8</th>
<th>16</th>
<th>5</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>GPA</td>
<td>78</td>
<td>80</td>
<td>75</td>
<td>94</td>
<td>89</td>
<td>92</td>
<td>80</td>
<td>84</td>
</tr>
</tbody>
</table>

(a) Find the equation for the line of best fit and the associated $r$-value. Round the linear coefficients to the nearest tenth and the $r$-value to the nearest hundredth. Then create a scatter plot with both the data and the line graphed. You do not need to reproduce that graph here unless your teacher asks you to.

(b) What is the value of the residual associated with the data point $(11, 94)$? Show the calculation that leads to your answer.

(c) Produce, using your calculator, the residual graph. It does not need to be exact, but show your WINDOW and the correct general location of the residuals.

(d) Why does this residual plot show a more appropriate linear model than the one in Exercise #1, even though the $r$-value is worse?
RESIDUALS

COMMON CORE ALGEBRA I HOMEWORK

1. Which of the following residual plots indicates a model that is most appropriate?

(1)  

(3)  

(2)  

(4)  

2. Which of the following residual plots would indicate the linear model used to produce it was an inappropriate choice?

(1)  

(3)  

(2)  

(4)  

3. A set of data is fit with linear regression. The equation for the best fit line was \( y = 5.2x + 18 \). If the observed value when \( x = 10 \) was \( y = 62 \), then which of the following represents the value of the residual for this data point?

(1) 52  
(2) –8  
(3) 8  
(4) –10
4. Physics students are performing a lab where they allow a ball to roll down a ramp and record the distance that it has rolled versus the time it has been rolling. The data for one such experiment are shown below.

<table>
<thead>
<tr>
<th>Time (sec)</th>
<th>0</th>
<th>0.5</th>
<th>1.0</th>
<th>1.5</th>
<th>2.0</th>
<th>2.5</th>
<th>3.0</th>
<th>3.5</th>
<th>4.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distance (ft)</td>
<td>0</td>
<td>0.4</td>
<td>1.5</td>
<td>3.2</td>
<td>5.6</td>
<td>8.5</td>
<td>12.6</td>
<td>17.2</td>
<td>22.8</td>
</tr>
</tbody>
</table>

(a) Determine the equation for the line of best fit. Round your coefficients to the nearest tenth. Also, determine the correlation coefficient. Round it to the nearest hundredth.

(b) Using your calculator, produce a scatter plot of this data set along with the line of best fit. Do your best job to sketch it below. Label the WINDOW you used.

(c) Calculate the residual for the data point (2.0, 5.6). Show your calculation below.

(d) Create a graph of the residuals using your calculator. Draw a sketch of the graph below, showing the WINDOW and the approximate location of the points. Make sure to turn off your other scatter plot and the line of best fit. Sketch the residual plot to the right.

(e) Is the linear model appropriate for this data set given the residual plot? Explain below.
UNIT #11

A FINAL LOOK AT FUNCTIONS AND MODELING

Lesson #1 – Function Transformations
Lesson #2 – Horizontal Stretching of Functions
Lesson #3 – Discrete Functions
Lesson #4 – Another Look at Linear and Exponential Models
Lesson #5 – Step Functions Revisited
Lesson #6 – Piecewise Linear Functions
Lesson #7 – Quadratic Models
Lesson #8 – Limits on the Accuracy of Our Models
We have transformed many functions this year by shifting them and stretching them. These transformations occur on a general basis and we will explore them in the next two lessons by looking almost exclusively at functions defined graphically. Still, we will rely heavily on function notation.

**Exercise #1:** The function \( y = f(x) \) is defined by the graph below. Answer questions based on this definition. Selected points are marked on the graph.

(a) Evaluate each of the following:
\[
\begin{align*}
    f(3) & = \\
    f(7) & = \\
    f(-4) & = \\
    f(-7) & = \\
\end{align*}
\]

(b) State the zeroes of \( f(x) \).

(c) Why is it impossible to evaluate \( f(9) \)?

(d) State the domain and range of \( f(x) \).

**Domain:**

**Range:**

O.K. Now that we have a bit of a feel for \( f(x) \) we are going to start to create other functions by transforming the function \( f \).

**Exercise #2:** Let’s now define the function \( g(x) \) by the formula \( g(x) = 2f(x) \).

(a) Evaluate each of the following. Show the work that leads to your answer. Remember, just follow the function’s rule.
\[
\begin{align*}
    g(-7) & = \\
    g(-4) & = \\
    g(3) & = \\
    g(7) & = \\
\end{align*}
\]

(b) How can you interpret the function rule in terms of the graph of \( f(x) \)?

(c) Sketch a graph of \( f(x) \) on the grid above in Exercise #1. Write down points that you know are on \( g(x) \) based on your answers to (a).

(d) State the domain and range of the function \( g(x) \).

**Domain:**

**Range:**
So, we see from the last exercise that when a function gets multiplied by a constant, all of the $y$-values get multiplied by the same constant. This has the effect of “stretching” a function.

**Vertical Stretch**

If the function $g(x)$ is defined by $g(x) = k \cdot f(x)$, then the graph of $g$ will be stretched (or compressed) depending on the value of $k$. If $k$ is negative, it will also reflect the function across the $x$-axis.

**Exercise #3:** A quadratic $f(x)$ is shown below. The function $g(x)$ is defined by $g(x) = -\frac{1}{2} f(x)$.

(a) Calculate the values of $g(0)$ and $g(3)$. Show your work.

Explain the effect of multiplying by $-\frac{1}{2}$.

(b) Sketch an accurate graph of $g(x)$ on the same grid as $f(x)$.

(c) State the range of $g(x)$.

Let’s do one final problem to see how well you understand what happens to the graph of a function when it has been multiplied by a constant.

**Exercise #4:** The function $f(x)$ is graphed as the bold curve shown below. Three other functions are all defined in terms of $f$ and are graphed as well. Label each curve with the appropriate function.

$$g(x) = \frac{1}{2} f(x)$$

$$h(x) = -f(x)$$

$$k(x) = 2f(x)$$
**FUNCTION TRANSFORMATIONS**  
**COMMON CORE ALGEBRA I HOMEWORK**

**FLUENCY**

1. Given the function \( f(x) \) shown in the table below, which of the following represents the value of \( g(4) \) given that \( g(x) = 5f(x) + 1 \)?

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>-8</td>
</tr>
<tr>
<td>3</td>
<td>-1</td>
</tr>
<tr>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>5</td>
<td>7</td>
</tr>
</tbody>
</table>

(1) 16  (3) 35  
(2) 40  (4) 0

2. The graph of \( y = x^2 \) is shown below in bold and labeled. Which of the following could be the equation of the graph shown in dashed?

(1) \( y = 2x^2 \)  
(2) \( y = -\frac{1}{2}x^2 \)  
(3) \( y = \frac{1}{3}x^2 \)  
(4) \( y = -4x^2 \)

3. The graph of \( f(x) \) is shown below in bold. Three other equations of functions are also given. Match each equation with the appropriate graph.

\[ g(x) = -f(x) \]
\[ h(x) = \frac{3}{2}f(x) \]
\[ k(x) = \frac{1}{2}f(x) \]

4. The function \( f(x) \) has \( x \)-intercepts of \(-3\) and \(5\) and a \( y \)-intercept of \(4\). If \( g(x) = 3f(x) \), then which of the following will be true about the graph of \( g(x) \)?

(1) It will have \( x \)-intercepts of \(-9\) and \(15\) and a \( y \)-intercept of \(12\).
(2) It will have \( x \)-intercepts of \(-3\) and \(5\) and a \( y \)-intercept of \(12\).
(3) It will have \( x \)-intercepts of \(-9\) and \(15\) and a \( y \)-intercept of \(4\).
(4) It will have \( x \)-intercepts of \(0\) and \(8\) and a \( y \)-intercept of \(7\).
5. The quadratic function \( f(x) = x^2 - 1 \) is shown graphed on the grid below. Two additional functions are defined as:

\[
  g(x) = 2f(x) \quad \text{and} \quad h(x) = f(x) + 2
\]

(a) Graph \( g(x) \) on the grid and label it. What is the effect of multiplying \( f(x) \) by 2?

(b) Graph \( h(x) \) on the grid and label it. What is the effect of adding 2 to \( f(x) \)?

6. Find equations for the functions \( g(x) \) and \( h(x) \) in terms of \( x \).

**APPLICATIONS**

6. A factory operated a printing press that produced pages of text at a rate that rises over the span of a 16 hour schedule, plateaus and then decreases. The rate can be modeled by the function \( R(t) \) shown.

   If the factory adds another printing press of the same size, it will now have a production rate of:

   \[ 2R(t) \]

   Graph the factory’s new rate on the same grid.

   What is the peak rate of the factory after it adds the second printing press? For how many hours does it maintain this peak rate?

**REASONING**

7. If both a vertical stretch and a vertical shift occurred to a function in the form of \( a \cdot f(x) + k \), which transformation occurred first? How can you tell?
In the last lesson we saw how multiplying a function by a constant stretched (or compressed) the function’s outputs, and thus its graph. This was a **vertical stretch** because it only affected the vertical (output) component of the function for a given input. In today’s lesson, we will see what happens to a function when you first manipulate its input.

**Exercise #1:** The function $f(x)$ is shown on the graph below. Selected points are shown as reference. The function $g(x)$ is defined by $g(x) = f(2x)$. Notice that the multiplication by 2 happens **before** $f$ is even evaluated. This is tricky!

(a) Find the values of each of the following. Carefully follow the rule for $g(x)$ and show your work.

$$g(2) = \quad g(3) =$$

$$g(-2) = \quad g(-4) =$$

$$g(0) = \quad g(-3) =$$

(b) Given the definition of $g(x)$, why can we **not** find a value for $g(4)$? Explain.

(c) State points that must lie on the graph of $g(x)$ based on your work in (a).

(d) Graph the function $g(x)$ based on your work from (b). Then, state the domain and range of both the original function, $f(x)$ and our new function $g(x)$. What remained the same? What changed?

**Original Function:** $f(x)$  
**New Function:** $g(x)$

<table>
<thead>
<tr>
<th>Domain</th>
<th>Range</th>
<th>Domain</th>
<th>Range</th>
</tr>
</thead>
</table>

(e) Describe what happened to the graph of $f(x)$ when we multiplied the function’s input by 2.
Notice how the **horizontal stretch** worked almost counter to what we would have thought. In other words, when we multiplied the $x$-value by 2, it **compressed** our graph by a factor of 2. The opposite would also occur.

**Exercise #2:** The function $f(x) = |x| - 3$ is shown on the graph below. The function $g(x)$ is defined by the formula $g(x) = \left| \frac{1}{2}x \right| - 3$.

(a) Use your graphing calculator to produce a table of values for $g(x)$ and graph it on the grid to the right.

(b) What was the effect on the graph of $f(x)$ when we multiplied the input by $\frac{1}{2}$?

We can certainly combine the effects of both a vertical stretch and a horizontal stretch. This is harder, but if you can identify the various transformations, then the new function’s graph can often be produced from the older function’s fairly easily.

**Exercise #3:** The graph of $f(x)$ is shown on the grid below. A new function $h(x)$ is defined by:

$$h(x) = 2f(3x)$$

(a) Evaluate $h(1)$. What point must lie on the graph of $h(x)$ based on this calculation?

(b) Describe the transformations that must be done to the graph of $f(x)$ to produce the graph of $g(x)$.

(c) Graph $g(x)$ by plotting the three major points.
HORIZONTAL STRETCHING OF FUNCTIONS
COMMON CORE ALGEBRA I HOMEWORK

FLUENCY

1. The function \( f(x) \) is shown graphed on the axes below with selected points highlighted. Two additional functions are defined as:

\[
g(x) = f(2x) \quad \text{and} \quad h(x) = 2f(x)
\]

Graph both \( g(x) \) and \( h(x) \) on the same grid and label them.

State the domain of \( g(x) \) only:

2. The quadratic function \( f(x) \) is shown graphed to the right. Three other functions are defined below with equations based on \( f(x) \). Label each graph with its appropriate function.

\[
g(x) = -f(x) \\
h(x) = f(2x) \\
k(x) = f\left(\frac{1}{2}x\right)
\]

3. Which of the following formulas would indicate that the graph of \( h(x) \) was stretched in the horizontal direction by a factor of 3?

(1) \( h(3x) \)  
(2) \( h\left(\frac{1}{3}x\right) \)  
(3) \( h(x) + 3 \)  
(4) \( 3h(x) \)
4. The parabola \( f(x) = x^2 - 16 \) is shown graphed on the grid below with certain points highlighted. The function \( g(x) \) is given by \( g(x) = f(2x) \).

(a) What is the range of the function \( f(x) \)?

(b) State the zeroes of \( f(x) \).

(c) The function \( g(x) \) will have the equation \( g(x) = (2x)^2 - 16 \). Using your calculator, create a graph of \( g(x) \) on the grid given.

(d) State the zeroes of \( g(x) \). Why does this answer make sense in light of (b)?

**REASONING**

5. The function \( f(x) \) is shown below. Another function is defined by the formula:

\[
g(x) = f(2x) + 3
\]

(a) Evaluate each of the following. Show your work.

\[
g(-3) = \quad g(-1) = \quad g(2) = \quad g(3) =
\]

(b) Plot a graph of \( g(x) \) based on (a).

(c) What two transformations occurred to the graph of \( f(x) \) to produce the graph of \( g(x) \)? State them and their order.
We have done a lot of modeling this year. Each time we used a function to describe the relationship between two quantities the input variable (typically $x$ or $t$) was either continuous or discrete. A non-rigorous set of definitions for continuous and discrete is given below:

**CONTINUOUS VERSUS DISCRETE VARIABLES**

A _continuous variable/function_ takes on all real number values between its extremes.

A _discrete variable/function_ takes on isolated or unconnected values between its extremes.

In this lesson we will concentrate on _discrete functions_ because most of the graphing we have done has been of _continuous functions_.

**Exercise #1:** Miranda has a lemonade stand where she is selling cups of lemonade for $0.50 per cup.

(a) Fill out the table below for the amount of money, $m$, that Miranda makes as a function of the number of cups, $c$, that she sells.

<table>
<thead>
<tr>
<th>Cups, $c$</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Money, $m$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(b) Create an equation that finds the money made, $m$, as a function of the number of cups, $c$, sold.

(c) Explain why this is an example of a _discrete function_.

(e) How many cups of lemonade would Miranda need to sell in order to make exactly $30?

(f) Explain why Miranda cannot make exactly $28.75.

(d) Graph this function on the grid below.
So, **discrete functions** are **characterized** by **domains** (inputs) that **realistically** contain only certain types of numbers, typically **whole numbers**.

**Exercise #2:** In each of the following cases, two variables are related by a function. In each situation, determine whether the function is **continuous** or **discrete**. Explain your thinking.

(a) A person who is driving at a constant speed of 62 miles per hour has a distance traveled, \(d\), given as a function of time, \(t\), in hours as \(d = 62t\).

(b) Franklin is selling candy bars to raise money for the Drama Club. Each candy bar costs $2.50. The money he raises, \(m\), as a function of the number of candy bars, \(b\), he sells is given by \(m = 2.50b\).

(c) A teacher imposes a one-half multiplicative penalty each day that an assignment is turned in late. The total credit, \(c\), that a student can earn based on the number of days it is late is given by \(c = 100 \cdot \left(\frac{1}{2}\right)^d\).

(d) A bathtub is draining at a rate of 3.2 gallons per minute from an initial volume of 164 gallons. The volume, \(V\), of water left in the bathtub after \(m\) minutes is given by \(V = 164 - 3.2m\).

Phenomena that are discrete often have ramifications when it comes to realistic solutions to modeling problems. Consider an example that compares texting plans.

**Exercise #3:** Malik is trying to compare texting plans for two cell phone companies. His options are given below.

**Option A:** A monthly charge of $12.50 and each text costs $0.02.

**Option B:** No monthly charge, but a charge of $0.05 per text.

(a) Write equations that give the total monthly cost, \(c\), based on the number of text’s made, \(n\), for both options.

Option A:

Option B:

(b) Why will Malik not be able to find a number of texts where the two plans charge an equal monthly amount?

(c) Even though the solution to (b) is not **viable**, it still might be helpful in thinking about the two cell phone plans. What information does it provide?
**APPLICATIONS**

1. In each of the following cases, two variables are related by a function. In each situation, determine whether the function is **continuous** or **discrete**. Explain your thinking.

   (a) The height, \( h \), of an object above the ground can be modeled as a function of time, \( t \), by the equation \( h = 200 - 16t^2 \).

   (b) The cost \( C \) of a charter bus trip depends on the number of people, \( n \), who go on the trip. This dependence can be shown in the equation \( C = 22.50n \).

2. Which of the following would be an example of two variables related with a discrete function.

   (1) The volume of water in a swimming pool and the amount of time it has been filling.
   (2) The cost of buying pens and the number of pens purchased.
   (3) The area of a square garden and the length of the side of the garden.
   (4) The mean temperature of a planet and its distance from the sun.

3. Maxwell is attempting to determine the volume of a penny in cubic centimeters. He does an experiment where he drops pennies into water and records the volume, in milliliters. The data is shown below.

<table>
<thead>
<tr>
<th>Number of Pennies</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Volume (mL)</td>
<td>10.5</td>
<td>10.8</td>
<td>11.1</td>
<td>11.4</td>
<td>11.7</td>
<td>12.0</td>
<td>12.3</td>
</tr>
</tbody>
</table>

   (a) Explain why the volume is a discrete function.
   (b) Graph the data from the chart on the grid below.
   (c) Write an equation for the volume, \( v \), as a function of the number of pennies, \( p \), placed in the water. This is a discrete linear function.
   (d) One milliliter is equivalent to one cubic centimeter. What is the volume of one penny in cubic centimeters?
4. Shana is trying to make sure that a local farm has enough bags of horse feed to last the week. She knows she wants to have 3 bags of feed per horse and a reserve of 8 bags as well.

(a) Determine an equation for the number of bags, \( b \), that Shana should plan on as a function of the number of horses, \( h \), present on the farm.

(b) Create a graph of your equation on the grid below.

(c) Using your equation from (a), how many bags of feed should Shana keep stocked if the farm has 15 horses?

(d) Using your equation from (a), how many horses can be on the farm if Shana has 62 bags of feed?

5. An amusement park models the amount of wait-time, \( W \), in minutes for a ride based on the number of people, \( n \), standing in line. The equation they determine is:

\[ W(n) = 0.4n + 12 \]

(a) Explain why this is an example of a discrete function.

(b) Interpret the fact that \( W(10) = 16 \). In other words, what does this mean in terms of the scenario being modeled.

(c) If the park estimates that the wait time is 45 minutes, then how many people must be standing in line for this exact wait time? Why is this not a viable solution?
In this lesson we will be looking at linear and exponential models again and trying to understand when it makes more sense to use one, rather than the other.

**Exercise #1:** A tank is being filled up with water. At $t = 0$, we know that the tank holds 150 gallons of water, and after one hour ($t = 1$), it holds 180 gallons of water.

(a) Assuming that the volume of water in the tank, $V$, is a linear function of time, $t$, in hours, find a formula for $V$.

(b) By what percent did the volume of water increase from $t = 0$ to $t = 1$?

(c) Based on (b), write an exponential function for $V$ as a function of the time, $t$, that it has been filling.

(d) After the tank has been filling for 10 hours, the volume is now at 500 gallons. Which model, the linear or exponential, better fits this data point?

It is good to be able to look at a variety of different forms of functions and determine what type of function best fits the information. Let’s take a look at that in the next exercise.

**Exercise #2:** The area, $A$, of an oil spill is increasing and scientists are trying to model it as a function of time so that they can predict when it reaches certain critical sizes. They measure the data and find the following.

<table>
<thead>
<tr>
<th>$t$ (days)</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$ (square miles)</td>
<td>3.5</td>
<td>4.4</td>
<td>5.5</td>
<td>6.8</td>
<td>8.5</td>
</tr>
</tbody>
</table>

(a) Explain why a linear function would not fit this data well.

(b) An exponential function of the form $A = a(b)^t$ does model this data well. Select which of the following would be the most appropriate values for $a$ and $b$:

\[
a: \quad 2.6 \quad 3.5 \quad 6.4 \quad 8.5
\]

\[
b: \quad 0.92 \quad 1.18 \quad 1.25 \quad 1.48
\]
We want to be very sure that we understand the various constants or **parameters** that come up in linear and exponential functions. Because these parameters **always** have a meaning in a physical situation.

**Exercise #3:** Two scenarios are modeled using in (a) a linear function and in (b) an exponential function. In each case interpret the parameters that help define the functions.

(a) Plant managers at a local tire factory model the cost, $c$, in dollars of producing $n$-tires in a day by the equation:

$$c(n) = 6.50n + 1,245$$

Interpret the parameter values of 6.50 and 1,245. Include units in your answer.

(b) Biologists model the population, $p$, of lactic acid bacteria in yogurt as a function of the number of minutes, $m$, since they added the bacteria using the equation:

$$p(m) = 135(1.28)^m$$

Interpret the parameter values of 135 and 1.28. Include units in your answer.

We can also work with approximate models based on **regression work** with **bivariate data sets**.

**Exercise #4:** The rate that soil can absorb water during a rain storm decreases over time as the rain continues. The table below gives the rate $y$, in inches per hour, that water can be absorbed as a function of the number of hours that rain has been falling, $x$.

<table>
<thead>
<tr>
<th>$x$ (hours)</th>
<th>0</th>
<th>1</th>
<th>3</th>
<th>5</th>
<th>6</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$ (inches/hour)</td>
<td>12.3</td>
<td>9.4</td>
<td>5.9</td>
<td>3.5</td>
<td>2.7</td>
<td>1.1</td>
</tr>
</tbody>
</table>

(a) Find the linear correlation coefficient for this data set? Round to the nearest thousandth. Why is it negative? Does this indicate a strong negative correlation or a weak negative correlation? Explain.

(b) Produce a rough sketch of the residual plot for this data set based on (a). Does the residual plot indicate that a linear model is appropriate? Explain.

(c) Find the exponential regression equation for this data set. Round both parameters to the nearest hundredth.

(d) Produce a rough sketch of the residual plot for this data set based on (c). Does this plot indicate a more appropriate model?
ANOTHER LOOK AT LINEAR AND EXPONENTIAL MODELS
COMMON CORE ALGEBRA I HOMEWORK

APPLICATIONS

1. For each of the following modeling scenarios, determine the equation asked for.

   (a) Nate is driving away from Albany at a constant rate of 62 miles per hour. If he starts 22 miles from Albany at \( t = 0 \), determine an equation for Nate’s distance, \( D \), from Albany after \( t \)-hours.

   (b) A bacteria culture is doubling in size every day. If the bacteria culture starts at 5,200, write an equation for its population size, \( p \), as a function of the number of days, \( d \), since it started.

2. The population of Arlington High School grew quickly at the turn of the millennium. The table below shows the population in 2000, 2001, and 2010.

<table>
<thead>
<tr>
<th>Year</th>
<th>2000</th>
<th>2001</th>
<th>2010</th>
</tr>
</thead>
<tbody>
<tr>
<td>( t )</td>
<td>0</td>
<td>1</td>
<td>10</td>
</tr>
<tr>
<td>Population</td>
<td>2600</td>
<td>2704</td>
<td>3610</td>
</tr>
</tbody>
</table>

   (a) Based on the first two years, write a linear equation for the population, \( P \), of Arlington High School \( t \)-years after the year 2000.

   (b) Based on the first two years, write an exponential equation for the population, \( P \), of Arlington High School \( t \)-years after the year 2000.

   (c) Based on the population of Arlington in 2010, which model seems to be a better fit for the population trend? Justify your choice.
3. eMathInstruction is keeping track of the number of views on a newly released math lesson screencast. They record the total number of views as a function of the number of days since it launched, with the launch day being \( x = 0 \).

<table>
<thead>
<tr>
<th>Days, ( x )</th>
<th>0</th>
<th>3</th>
<th>5</th>
<th>8</th>
<th>13</th>
<th>17</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>Views, ( y )</td>
<td>56</td>
<td>92</td>
<td>130</td>
<td>212</td>
<td>486</td>
<td>920</td>
<td>1,530</td>
</tr>
</tbody>
</table>

(a) The residual graph for the line of best fit is shown below. Indicate whether this statistic indicates that a linear function is appropriate to model the data.

(b) The residual graph created when doing an exponential fit for this data is shown below. Does this statistic indicate a better or worse fit than the linear model from (a)? Explain.

4. The regression equations for the two types of models were found using the data from Problem #3. They are as follows:

**Linear:**

\[ y = 68x - 157 \]

**Exponential:**

\[ y = 56(1.18)^x \]

(a) How can you interpret the parameter 68 in the linear model in terms of the views of the website?

(b) How do you interpret the parameter 1.18 in the exponential model in terms of the views of the website?

(c) Why is the interpretation of the \(-157\) in the linear model unreasonable or nonviable?
Step functions, or ones whose outputs stay constant and then jump to a new constant value, are critical to a number of real world applications. Many times these types of functions arise in the areas of business.

**Exercise #1:** An electrician works at a job site at a rate of $40 per hour or any portion of an hour. In other words, he will charge you $40 as soon as he comes up to the first hour, and then $40 for the second hour, etcetera.

(a) Graph the amount the electrician charges, \( c \), in dollars as a function of the number of hours he works.

(b) How much does he charge for working 3.5 hours? Circle the point on the graph the shows this answer.

Step functions are rather simple because they consist of multiple horizontal lines. When reading their formula definitions, it is important to pay attention to the domain intervals.

**Exercise #2:** A step function is defined using the piecewise formula

\[
f(x) = \begin{cases} 
2 & 0 \leq x < 3 \\
5 & 3 \leq x < 5 \\
-4 & 5 \leq x \leq 10 
\end{cases}
\]

(a) Evaluate the following:

\[
f(2.7) = \quad f(5) = \\
f(3.5) = \quad f(0) = 
\]

(b) Graph \( f(x) \) on the grid to the right.

(c) State the domain and range of this function.

Domain: 
Range:
Step functions are used in engineering to signify when we have changes in constant rates. These functions can give rise then to piecewise linear functions.

**Exercise #3:** A pumping station is draining a reservoir with a set of pumps that drain the water at a rate of 250 gallons per hour. After 5 hours, additional pumps are turning on such that they pump at an overall rate of 600 gallons per hour for the next 7 hours.

(a) Draw a graph of the pump rate function on the grid provided.

(b) How many total gallons of water are pumped out of the reservoir over the 12 hour period? Show the calculations that lead to your answer.

(c) The reservoir originally contains 8,250 gallons of water. How much does it contain after 5 hours if water is only pumped out? Show the work that leads to your answer.

(d) Engineers want to turn off the pumps when the reservoir reaches a level of 2,000 gallons. Will they need to turn the pumps off during this 12-hour time period? Show evidence to support your yes/no answer.

(e) Assuming engineers do not turn off any pumps, how many total hours will it take, to the nearest tenth of an hour, to drain the reservoir of all of its water?
**STEP FUNCTIONS**

**COMMON CORE ALGEBRA I HOMEWORK**

**FLUENCY**

1. Consider the step function given by \( f(x) = \begin{cases} 
5 & 0 \leq x < 4 \\
1 & 4 \leq x < 8 \\
-3 & 8 \leq x \leq 12 
\end{cases} \), which actually does a half-way decent job of modeling downward steps.

   (a) Graph \( f(x) \) on the grid provided.

   (b) State the range of this function.

   (c) Does \( f(x) \) have any zeroes? Explain.

2. The step function \( g(x) \) is shown on the grid to below. Answer the following questions.

   (a) Evaluate each of the following:

   \[
   f(-4) = \quad f(-2) = \\
   f(2) = \quad f(5) = 
   \]

   (b) Ji Hwan states that the range of this function is \(-3 \leq y \leq 4\). Is he correct? Why or why not.

   (c) Write an equation for this step function:

   \[
g(x) = \begin{cases} 
\text{ } & \\
\text{ } & 
\end{cases}
\]
APPLICATIONS

3. When kimchi is made, it is initially fermented for the first 3 days at a temperature of 70 degrees Fahrenheit and then immediately moved to a temperature of 50 degrees Fahrenheit for another 3 days after which it is put in a 35 degree refrigerator for 6 days.

The Fahrenheit temperature, \(F\), of the kimchi can be modeled over time, \(t\), in days with the equation below. Graph the kimchi’s temperature on the grid provided.

\[
F(t) = \begin{cases} 
70 & 0 \leq t < 3 \\
50 & 3 \leq t < 6 \\
35 & 6 \leq t \leq 12 
\end{cases}
\]

4. Stewart International Airport in Newburgh, New York charges for parking the way many airports do, by the partial hour. Their short-term parking rates are shown below.

(a) Explain why the total amount you will pay for parking at Stewart is a step function based on the number of hours you’ve parked?

(b) How much would you have to pay if you parked for 5 hours and 22 minutes? Show how you determined your answer.

(c) After how many hours of parking will you hit the maximum charge of $30? Explain your reasoning.
We modeled with piecewise functions back in Unit #3. In today’s lesson we will work specifically with piecewise linear functions, or those that are comprised of linear segments. These are particularly helpful in modeling certain situations, especially with motion.

**Exercise #1:** Mateo is walking to school. It’s a nice morning, so he is moving at a comfortable pace. After walking for 9 minutes, he is 6 blocks from home. He stops to answer a text on his phone from his mother. After 5 minutes standing still, he walks home quickly in 6 minutes to get a paper he forgot for school. We are going to model Mateo’s distance from home, \( D \), in blocks as a function of the time, \( t \), in minutes since he left.

(a) Draw a graph of Mateo’s distance from home on the grid provided.

(b) Determine a formula for the distance he is from home, \( D \), over the time interval \( 0 \leq t \leq 9 \).

(c) Determine a formula for the distance he is from home, \( D \), over the time interval \( 9 \leq t \leq 14 \).

(d) The trickiest part of this modeling will be to determine the linear equation for the distance, \( D \), on the time interval \( 14 \leq t \leq 20 \). Pick two points on this line and form an equation in the form \( D = mt + b \).

Piecewise linear functions are more complex function rules. One way or another, though, they fit the standard definition of a function, i.e. for every value in the domain (\( x \)) there is only one value in the range (\( y \)).

**Exercise #2:** Consider the function defined by:

\[
f(x) = \begin{cases} 
2x + 4 & \text{if } -4 \leq x \leq 1 \\
6 - x & \text{if } 1 < x \leq 5 
\end{cases}
\]

(a) Graph the function \( f(x) \) by graphing each of the two lines.

(b) State the range of the function \( f(x) \).
Piecewise linear functions can often have horizontal components as well as slanted components. They will obviously never have vertical components (or they wouldn’t be functions). Let’s see if we can translate from a graph to a piecewise equation.

**Exercise #3:** The piecewise linear function \( f(x) \) is shown graphed below.

(a) Find the slope of each of the line segments:

\[ \overline{AB} : \quad \overline{BC} : \quad \overline{CD} : \]

(b) Now find the equation of the line that passes through each of the following pairs of points in \( y = mx + b \) form where applicable. How can you find the \( y \)-intercepts by using the graph?

\[ \overline{AB} : \quad \overline{BC} : \quad \overline{CD} : \]

(c) Write the formal piecewise definition for this function.

(d) Find the one zero of the function algebraically by setting the formula for this function that applies from \(-6 \leq x \leq -2\) equal to zero and solving.

(e) Why does setting the formula for this function that applies from \(2 \leq x \leq 6\) equal to zero not produce a viable zero of the function?

(f) What parameter in the piecewise linear model indicates that the function is decreasing between \(x = 2\) and \(x = 6\)? Explain your choice.
PIECEWISE LINEAR FUNCTIONS
COMMON CORE ALGEBRA I HOMEWORK

FLUENCY

1. Given the function \( f(x) = \begin{cases} 2x + 6 & x < -1 \\ -4x + 8 & x \geq -1 \end{cases} \), answer the following questions.

   (a) Evaluate each of the following function values. Carefully pay attention to which of the formulas applies.

   \[ f(2) = \quad f(0) = \]

   \[ f(-5) = \quad f(-1) = \]

2. Given the piecewise function \( g(x) = \begin{cases} \frac{1}{2}x + 6 & x < 0 \\ 4x + 1 & x \geq 0 \end{cases} \), what is the average rate of change over the interval \(-2 \leq x \leq 1\)?

   (1) \( \frac{1}{2} \)  
   (3) \(-3\)

   (2) 0  
   (4) 4

3. On the graph below, sketch the function \( h(x) = \begin{cases} -2x - 6 & -6 \leq x < 0 \\ \frac{1}{2}x - 6 & 0 \leq x \leq 4 \end{cases} \).

   (a) Graph \( h(x) \) on the grid.

   (b) State the range of \( h(x) \).

   (c) What values of \( x \) solve \( h(x) = 0 \)?
APPLICATIONS

4. A substantial snowstorm is hitting the Northeast region and is predicted to snow at a rate of 2 inches per hour for the first three hours of the storm. The storm is supposed to pause for three hours and then resume at a rate of one-half inch per hour for the next four hours. The depth, \( D \), of the storm is the total number of inches of snow that has fallen at a given time.

(a) How many hours is the snow storm?

(b) How many total inches of snow fell? Show the calculations that lead to your answer.

(c) Graph the snow depth as a function of time since the storm began for the length of the storm.

(d) Determine a piecewise linear function for \( D \) as a function of the number of hours, \( t \), since the storm began. There should be three formulas. The first two should be relatively simple, while the third might take some additional thinking.

REASONING

5. The function \( f(x) = \begin{cases} 2x - 8 & x < 0 \\ \frac{x}{2} + 5 & x \geq 0 \end{cases} \) has no zeroes even though each individual line, i.e. \( y = 2x - 8 \) and \( y = \frac{x}{2} + 5 \) each have a zero. Why does \( f(x) \) lack zeroes?
QUADRATIC MODELING
COMMON CORE ALGEBRA I

Physical scenarios that involve quadratic functions occur naturally in physics, economics, and a variety of other fields. Typically, the science behind these phenomena are beyond the scope of this course, so our quadratic modeling is less sophisticated than our linear or exponential. We will take a final look, with our modeling, of some scenarios that lend themselves well to these functions.

Exercise #1: Projectiles that are fired vertically into the air have heights that are quadratic functions of time. A projectile is fired from the top of a roof. It’s height, in feet above the ground, after \( t \)-seconds is given by the function:

\[ h(t) = -16(t - 2)^2 + 144 \]

(a) Evaluate \( h(0) \). Using proper units, explain the physical significance of this answer.

(b) Determine algebraically the time when the ball hits the ground.

(c) Create a graph of \( h(t) \) on the grid provided.

(d) What is the maximum height that the projectile reaches and at what time does it reach this height? Do you see this answer in the vertex form of the parabola?

Exercise #2: Popcorn has an optimal temperature at which it pops. Food engineers at Perpetual Popping study the percent of popcorn kernels that pop at a certain temperature. Their data is shown in the table below.

<table>
<thead>
<tr>
<th>Temp, ( t )</th>
<th>385</th>
<th>410</th>
<th>440</th>
<th>490</th>
<th>510</th>
<th>530</th>
</tr>
</thead>
<tbody>
<tr>
<td>Percent, ( P )</td>
<td>38</td>
<td>68</td>
<td>78</td>
<td>65</td>
<td>45</td>
<td>18</td>
</tr>
</tbody>
</table>

(a) Why does a quadratic model seem reasonable given the data in the table?

(b) If the engineers model the percent popped, \( P \), by the equation \( P = -\frac{1}{100}(t - 450)^2 + 82 \), then at what temperature is the greatest percent of popcorn popped? What is the greatest percent?
You can create some quadratic models just on your own from simple geometric ideas like perimeter and area. Let's do one last modeling problem that involves these two simple concepts.

**Exercise #3:** Shana is creating a garden that has three equal sized rectangles separated by wire fencing. She has 160 feet of fencing and wants to construct the garden as shown below. Shana decides to designate the overall width of the rectangle as $x$ and the overall length as $y$, as shown on the diagram.

(a) How much area would the garden contain, in square feet, if the width, $x$, was 10 feet? Show the calculations that lead to your answer.

(b) Write a formula for the overall area, $A$, of the garden in terms of $x$ and $y$. This should be a very simple formula.

(c) Write an equation for the relationship between the width, $x$, and the length, $y$, based on the fact that there is 160 feet of fencing. Solve this equation for $y$.

(d) Find an equation for the area, $A$, only in terms of the width, $x$.

(e) Using your calculator, sketch a graph of the area function you found in (d).

(f) What is the maximum area that Shana can enclose with the 160 feet of fencing? What dimensions should she use?
QUADRATIC MODELING
COMMON CORE ALGEBRA I HOMEWORK

APPLICATIONS

1. Physics students are modeling the height of an object dropped from the top of a 90 foot tall building. It is let go at \( t = 0 \) seconds and using photography the students are able to measure, accurate to the nearest tenth of a foot, the height that the object is above the ground every half-second. The data is shown below.

<table>
<thead>
<tr>
<th>( t ) (sec)</th>
<th>0</th>
<th>0.5</th>
<th>1.0</th>
<th>1.5</th>
<th>2.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>( h ) (ft)</td>
<td>90</td>
<td>86.1</td>
<td>74.2</td>
<td>54.5</td>
<td>26.8</td>
</tr>
</tbody>
</table>

(a) Given the scatterplot shown to the right, draw a quadratic of best fit by hand through the data. Extend your quadratic until it hits the \( x \)-axis.

(b) Students in the class approximate the equation of the quadratic of best fit by:

\[ h = -16t^2 + 90 \]

Calculate the residual from this model at \( t = 2 \) seconds. Show the work that leads to your answer.

(c) Use the students’ model above to determine algebraically the time, \( t \), when the ball hits the ground. Show your work and round to the nearest tenth. How does this answer compare with where you drew the zero on the graph?

2. The Fahrenheit temperature of a chemical reaction decreases over time, measured in minutes, and then increases according to the function:

\[ F(t) = \frac{1}{2}(t-8)^2 + 72 \]

(a) For the function above, \( F(0) = 104 \). Interpret what this means in terms of the chemical reaction.

(b) What is the minimum temperature reached during the reaction and at what time does it reach it?
3. The price of a stock rose and then fell in the span of 10 days of trading. Its price at various points in time since it was first offered is given in the table below.

<table>
<thead>
<tr>
<th>Day, $d$</th>
<th>0</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Price, $p$</td>
<td>$30.50$</td>
<td>$36.50$</td>
<td>$38.75$</td>
<td>$36.75$</td>
<td>$30.75$</td>
<td>$20.50$</td>
</tr>
</tbody>
</table>

(a) Explain why a quadratic function will model this data better than a linear or exponential function.

(b) If a quadratic function of the form

$$p = a(d - h)^2 + k$$

is used to model the price, $p$, of the stock as a function of the day, $d$, then give values for $h$ and $k$. Justify your choices.

(c) Which of the following value of $a$ would be the best choice for the model given your answers to (b)? Justify your choice.

$$a = -2 \quad a = -\frac{1}{2} \quad a = \frac{1}{2} \quad a = 2$$

4. A farm is creating fenced in pens and wants to lay out the pens in the following rectangular configuration where the width of the pens is given by the variable $x$ as shown. Engineers have only 90 feet of fencing to surround and divide the pens and have created the following equation for the total area enclosed, in square feet, based on the width of $x$:

$$A = 45x - 3x^2$$

(a) Determine the zeroes of this quadratic by factoring.

(b) How can you use your answers to (a) to help determine the $x$-value where the maximum area will occur?

(c) Find the maximum area of the pen. Show the calculation that gives your answer.
LIMITS TO ACCURACY OF OUR MODELS
COMMON CORE ALGEBRA I

Most mathematical models of real world phenomena contain errors. It is rare that we can predict the outcome to almost any event with 100% confidence (more on that in the statistics of Algebra II). Knowing what introduced error is important to consider as we conclude the lessons of this course. Let’s start by investigating a very simple mathematical model that you should feel comfortable with.

Exercise #1: Mia is trying to calculate the area of her closet so she can purchase wood flooring. She measures the width and length and rounds to the nearest tenth of a meter. She found the length to be 2.7 meters and the width to be 1.4 meters.

(a) Calculate the area of the rectangular floor. Include proper units.  
(b) Why does it not make sense to leave this answer accurate to the nearest hundredth? Write down a proper level of precision for the area. Include units.

Generally speaking, without getting too deep into what scientists refer to as significant figures, the limitations on any calculation or prediction will be limited by the least precise input to the model.

CHOOSING YOUR PRECISION LEVEL

The calculation of an output to a model should be rounded to the level of accuracy of the least accurate input to the model.

Exercise #2: The weight of newborns for a day were recorded at a local hospital. The weights were rounded to the nearest tenth of a pound. They are as follows:

6.2, 8.4, 5.6, 10.1, 7.4, 8.7, 9.3, 6.8, 7.5

Calculate the mean and the standard deviation of this data set. Include appropriate units for both results and round to appropriate levels of accuracy.
**Exercise #3:** Jonathan knows that if a projectile is fired from a height of exactly 3 meters above the ground at an initial speed of 24 meters per second, then its height, $h$, in meters above the ground after $t$ seconds will be given by the formula:

$$h = -4.9t^2 + 24t + 3$$

Jonathan starts a timer and takes a picture at a time when the ball is in the air. The picture records the time-signature, i.e. when the picture was taken, to be 1.7 seconds after it was launched, rounded to the nearest tenth of a second.

(a) Use the function above to determine the height of the projectile at $t = 1.7$ seconds. Do not round your answer. 

(b) Why should Jonathan not report the height of the projectile to the level of accuracy given in (a)? What should be the proper answer (with units)?

**Exercise #4:** Water control engineers are keeping track of the volume of water in a local storage facility. They measure the initial amount of water to be 362 gallons, to the nearest gallon. Water is being withdrawn at a rate of 12.8 gallons per minute, to the nearest tenth of a gallon per minute.

(a) Write a formula for the volume of water, $V$, left in the reservoir as a function of time, $t$, in minutes that the water has been draining.

(b) Engineers would like to know how much water is in the reservoir at 7 minutes. Determine the volume and use an appropriate level of precision.

**Exercise #5:** A radioactive material decays such that 5% of it is lost every hour. Scientists take a small portion of the material and weigh it to be 24.8 grams, to the nearest tenth of a gram. Develop an exponential formula, of the form $A = a(b)^t$ for the amount of material still radioactive after $t$-hours. Use your model to determine the amount still radioactive after 10.0 hours.
LIMITS TO ACCURACY OF OUR MODELS
COMMON CORE ALGEBRA I HOMEWORK

FLUENCY

1. Pick the best choice below to fill in the blank: The precision of any calculation based on inputs should be as precise as its ______________ input.

   - most precise
   - least precise

2. Given that each value in the data set below has been rounded, which of the following choices should we make for the mean of the data set.

   $6.1, 8.6, 4.35, 7.8, 2.71$

   (1) 5.912  (3) 5.9  
   (2) 6      (4) 5.91

APPLICATIONS

3. Jonathan is driving at 62 miles per hour, rounded to the nearest integer, away from Ashmore, Illinois. He started at $h = 0$ in Ashmore.

   (a) Write an equation for Jonathan’s distance from Ashmore, $d$, as a function of the number of hours he has been driving, $h$.

   (b) Determine Jonathan’s distance from Ashmore after driving for 2.7 hours, given that time has been rounded to the nearest tenth of an hour. Include units in your answer.

4. To be classified as a Large Egg, eggs must weigh between 2 and 2.25 ounces. A local hen farm selected 10 eggs they considered to be Large and weighed them to the nearest tenth of an ounce. Here is their data:

   $2.1, 2.3, 2.0, 2.1, 2.2, 2.5, 2.2, 2.3, 2.1, 1.9$

   (a) Determine the mean egg weight for this sample and the sample standard deviation. Include units and round to the correct precision level.

   (b) What percent of this data set should not have been classified as Large? Show the work that leads to your answer.
5. Engineers modeled the depth of water, in feet, in a reservoir as it is being drained by the equation:

\[ d(t) = 26(0.62)^t + 6 \]

where \( t \) is the number of hours it has been draining.

(a) According to the engineer’s model, what was the depth of the water at \( t = 0 \) hours?

(b) The engineers wanted to record the depth every tenth of an hour, but missed it at 3.7 hours, rounded to the nearest tenth of an hour. What was the depth according to their model?

6. Forrest biologists are trying to find a correlation between the height of maple trees and their diameters at ground height. They find the following data which has been rounded:

<table>
<thead>
<tr>
<th>Diameter, ( x ), in inches</th>
<th>4</th>
<th>10</th>
<th>13</th>
<th>20</th>
<th>24</th>
<th>32</th>
</tr>
</thead>
<tbody>
<tr>
<td>Height, ( y ), in feet</td>
<td>18</td>
<td>28</td>
<td>32</td>
<td>40</td>
<td>44</td>
<td>51</td>
</tr>
</tbody>
</table>

(a) Find the equation of the best fit line for this data set. Round all parameters to the nearest hundredth.

(b) Using your model, predict the height of a tree that has a diameter, rounded to the nearest inch, of 22 inches.

(c) The linear correlation coefficient for this data set is 0.99, rounded to the nearest hundredth. Does this indicate a strong, moderate, or weak positive association? Explain.

(d) Produce a rough plot of the residuals below. Does this indicate that a linear model is appropriate for this data set? Why or why not?

(e) In this model, we have a high \( r \)-value, but a residual graph that shows a pattern. In the statement below, circle one of the two words underlined (each time) to complete the statement.

A model with a high \( r \)-value may be very accurate appropriate but if its residual graph shows a definitive pattern then the model may not be very accurate appropriate.